Design of Exponentially Weighted Moving Average Scheme for Standardized Means

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ABSTRACT

Exponentially Weighted Moving Average (EWMA) scheme in monitoring process mean is favored because it remembers the past information and detects the small shifts in the mean of a sequence of independent normal variates. However the design procedure of the EWMA scheme was complex till Crowder (1989) presents a simplified scheme for monitoring process mean. Currently standardizing the variables and monitoring them jointly is widely practiced and many schemes are presented under joint quality monitoring. Therefore a need arises to design EWMA schemes for monitoring standardized process means which will easier the work of monitoring more than one process variable at a time. In this paper an alternative design of such a scheme is presented. The new scheme is easy to apply and is independent of sample size. The scheme parameters are the same for any process if the in-control average run length (ARL) and the shifts in means to be detected are the same. In industrial application this property is more beneficial for monitoring few variables in one display.

Keywords: Average run length, Exponentially weighted moving average, Process mean, Shift in mean.

1 Introduction

In quality control, monitoring of process mean by using Exponentially Weighted Moving Average (EWMA) was introduced by Roberts (1959). Subsequently different procedures of investigating process mean through EWMA were discussed by many authors (Robinson and Ho, 1978;, Crowder, 1987;, Chantraine, 1987; and Lucas and Saccicci, 1987). Further a simple design procedure of EWMA scheme for monitoring process mean was reported by Crowder (1989). Chang and Gan (1993) introduced the design procedure of monitoring variance using EWMA. These two design procedures are extensively used in monitoring of process mean and variance simultaneously and these joint monitoring techniques were presented by many authors (Gan, 1995;, Chen *et al*, 2001; and Chen *et al* 2004).

In industry, EWMA charts are designed for monitoring different process means and these charts are displayed separately. However the quality control engineer should keep on watching all the displays in parallel. As an alternative, use of standardized mean or variance for process monitoring has the advantage of having single chart display for monitoring several process parameters at a time. Further if the average run lengths (ARL) are same for the means to be monitored then the control limits for the chart will also be same. The use of standardized mean and variance and the advantages of this standardization are discussed by McCraken and Chakarborti (2013) with a revision of few joint monitoring schemes with standardized mean and variance. Chen and Cheng (1998) used standardized mean and variance for presenting max chart. The max chart combines two standardized statistics, one for mean and other for variance, by taking the maximum of the absolute values of the two statistics. Razmy (2010) used standardized mean and variance for introducing Shewhart distance scheme for joint quality monitoring. A technique of applying EWMA for standardized variable is useful because a single chart display can be used for monitoring several standardized process variables at a time while remembering the past information to detect the small shifts in the mean of a sequence of independent normal variates. This application can be extended for joint monitoring of standardized process mean and variance in advanced quality control. In this paper, a design of EWMA schmes for standartized mean is proposed with a simple design procedure. This new procedure is illustratred using an application example.

2 Scheme Development

Let X_{tj} denote a certain quality characteristic of a process where t is the sample number (t =1, 2, ..), j is the j^{th} unit of the sample and j = 1, 2, ..., n. It is assumed that X_{tj} 's are independently and identically normally distributed random variables with mean μ_0 and standard deviation σ_0. Let

$$\bar{X}_t = \frac{1}{n}\sum_{j=1}^{n} X_{tj} \tag{1}$$

be the t^{th} sample mean and

$$S_t^2 = \frac{1}{n-1}\sum_{j=1}^{n}(X_{tj} - \bar{X}_t)^2 \tag{2}$$

be the t^{th} sample variance. The sample mean for each sample is standardized as

$$U_t = \frac{\bar{X}_t - \mu_0}{\sigma_0/\sqrt{n}} \tag{3}$$

and when a process is in-control, U_t is standard normal random variable. The EWMA chart for monitoring the mean based on standardized sample mean is obtained by plotting

$$Q_t = (1 - \lambda_M)Q_{t-1} + \lambda_M U_t \tag{4}$$

against the sample number where λ_M is a positive smoothening constant such that $0 < \lambda_M \leq 1$. Q_0 is the $E(U_t)$ for the process and it is zero. Variance of Q_t is written as

$$Var(Q_t) = \frac{\sigma_0^2}{n}\left(\frac{\lambda_M}{2 - \lambda}\right)[1 - (1 - \lambda)^{2t}] \tag{5}$$

as reported in Crowder (1989). An out-of-control signal is issued if Q_t is greater than the upper control limit (UCL) H_M or Q_t is less than the lower control limit (LCL) $-H_M$. The control limits for the selected λ_M values are decided based on ARL that is the average number of samples taken before an out-of-control signal issued when the process is actually in–control. The control limits for selected λ_M values can be calculated based on the equations

$$UCL = \mu_0 + L\frac{\sigma_0}{\sqrt{n}}\sqrt{\frac{\lambda_M}{2-\lambda_M}\left[1-(1-\lambda_M)^{2t}\right]} \tag{6}$$

$$LCL = \mu_0 - L\frac{\sigma_0}{\sqrt{n}}\sqrt{\frac{\lambda_M}{2-\lambda_M}\left[1-(1-\lambda_M)^{2t}\right]} \tag{7}$$

where L is the number of standard deviation unit that is based on the selected ARLs such that

$$\frac{1}{ARL} = P(LCL > \mu_0 > UCL). \tag{8}$$

The calculated control limits for selected values of λ_M and ARL are displayed in Figure 1. These calculated control limits were rechecked using simulations. When

λ_M = 1, the value of Q_t depends only on the most recent sample *t*, just as the case of the Shewhart $\bar{X}$ bar chart (Shewhart, 1939). The optimal value of λ_M differs based on the shift in mean to be detected quickly and the ARL of the scheme. The shift in mean is normally said in number of standard deviation units away from the in-control mean. The new mean after a shift can be written as

$$\mu = \mu_0 + \Delta \frac{\sigma_0}{\sqrt{n}} \quad (9)$$

where Δ is the shift from the in-control mean in number of standard deviation units. To find an optimal λM for detecting a particular shift in mean quickly for a particular in-control ARL, the control limits for various λM values (λM = 0.025, 0.050,..., 0.095, 1.000) were found. Then for a particular shift, the λM value that gives the smallest out-of-control ARL is the optimal λM for that particular shift. The optimal λM for detecting different shifts in mean (Δ), for various values of ARL were found using simulations and these are displayed in Figure 2. In all cases simulations were run until the standard errors of the ARL's were less than 1 % of the pre-specified ARL's.

3 Scheme Design

The design procedure recommended by Crowder (1989) for his EWMA scheme is adopted for the design of this EWMA scheme for the standardized mean. The design strategy is to find λ_M for a given in-control ARL that minimizes the out-of-control ARL for a specified shift in the mean. To design this scheme, the following steps are recommended:

Step 1. Select the smallest acceptable in-control ARL for the scheme.

Step 2. Determine the magnitude of the shift in mean (Δ)to be detected quickly. Select the λ_M that gives the minimum out-of-control ARL at the shift selected.

Step 3. Given the value of λ_M determined in Step 2, find the control limit H_M, for the in-control ARL selected in step 1.

This scheme can then be implemented by plotting the monitoring statistics Q_t against the sample number t. A signal is issued if $Q_t > H_M$ or $Q_t < -H_M$. In Step 2, the value of λ_M can be read off easily from Figure 2 given the value of Δ and in-control ARL. Similarly using the value of λ_M obtained in Step 2, the control limit H_M can be read off from Figure 1.

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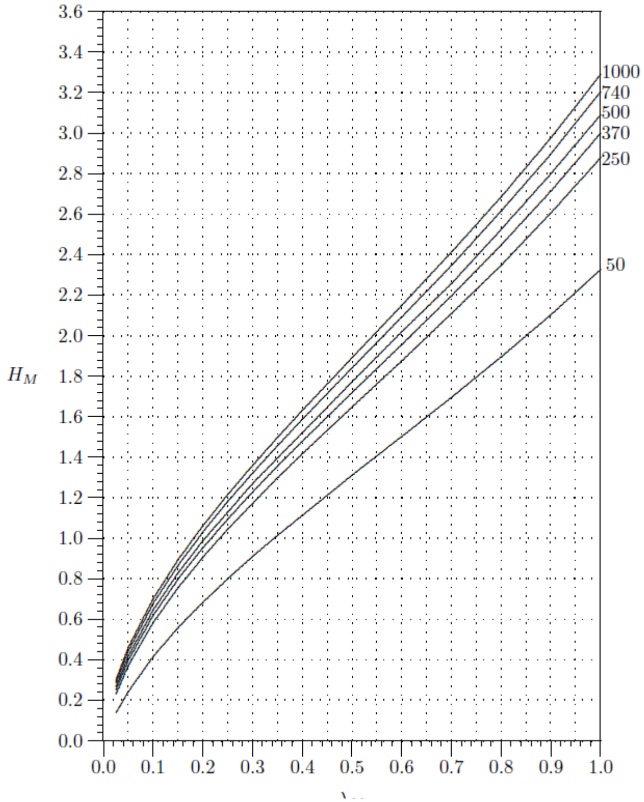

Figure1: Control limits for the EWMA mean scheme based on standardized sample means with in-control ARLs of 50, 250, 370, 500, 740 and 1000.

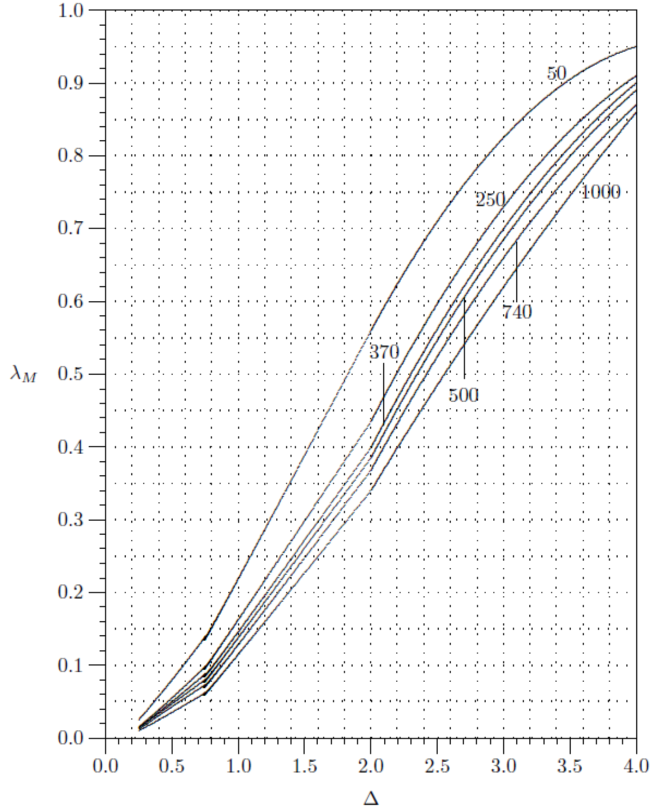

Figure 2: Optimal λ_M values for detecting various shifts in mean for the EWMA mean scheme based on standardized sample means with in-control ARLs of 50, 250,370, 500, 740 and 1000.

4 Application

If a scheme needs to have a quick detection (minimum out-of-control ARL) when there is one unit standard deviation shift in the mean ($\Delta = 1$) for the in-control ARL of 740, the following step can be carried out.

Step 1. Set the desired in-control ARL of the scheme that is ARL = 740.

Step 2. If $\Delta = 1$ shift in mean is to be detected quickly, the optimal λ_M = 0.125. (obtained from Figure 2 for ARL = 740 and $\Delta = 1$).

Step 3. From Figure 1, the control limit H_M is 0.66, based on λ_M = 0.125 and ARL=740.

Then the EWMA scheme for standardized means can be implemented by plotting the monitoring statistics Q_t against the sample number where a signal is issued if

$$-0.66 > Q_t > 0.66.$$

5 Conclusion

This paper provides an easy way of designing EWMA scheme for the standardized means. As means are standardized, in industrial application, several parameters can be monitored simultaneously in a single display. A big advantage of these schemes is that the scheme parameters are independent of the sample size which gives more freedom to the quality engineers in selecting sample size. Another advantage of these schemes is that the scheme parameters are the same for any process if the in-control ARL and the shifts to be detected are the same. For example the scheme parameters for a process with N (25, 3) with sample size 5 and for a process with N(200, 10) with the sample size 10 are the same if the in-control ARL and shifts in mean to be detected are the same. This design procedure will be helpful in developing combined monitoring schemes of parameters which is being a popular researchable area in quality control.

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