Group Acceptance Sampling Plans for Life Tests Based on Half Normal Distribution

B. Srinivasa Rao[1*], CH. Srinivas Kumar[2] and K. Rosaiah[3]
[1]Department of Mathematics & Humanities, R.V.R & J.C College of Engineering, Chowdavaram, Guntur-522 019, Andhra Pradesh, India.
[2]Department of Mathematics, K.L.University, Vaddeswaram, Guntur District-522 502, Andhra Pradesh, India.
[3]Department of Statistics, Acharya Nagarjuna University, Guntur- 522 010, Andhra Pradesh, India.
[*]Corresponding Author: boyapatisrinu@yahoo.com

Received: 11, February 2014 / Revised: 28, October 2014 / Accepted: 24, November 2014

ABSTRACT

In this article, a group acceptance sampling plan is developed based on truncated lifetimes when the lifetime of an item follows a half normal distribution. For a given group size, the minimum number of groups and the acceptance number required are determined for specified consumer's risk and the test termination time. The values of operating characteristic function for various quality levels are calculated and the minimum ratios of the true average life to the specified life at given producer's risk are obtained. The results are illustrated by examples.

Keywords: half normal distribution, group acceptance sampling plans, consumer's risk, operating characteristic (OC) function, producer's risk, truncated life test.

1. Introduction

Reliability study plays a vital role in the quality control analysis. On the basis of this study, an experimenter can save his time and cost to reach a result whether to accept the submitted lot or to reject it. An acceptance sampling plan is a scheme that establishes the minimum sample size to be used for testing. This becomes particularly important if the quality of product is defined by its lifetime. Often, it is implicitly assumed when designing a sampling plan that only a single item is put in a tester. However, in practice testers accommodating a multiple number of items at a time are used because testing time and cost can be saved by testing items simultaneously. The items in a tester can be regarded as a group and the number of items in a group is called the group size. For such a type of test, the determination of the sample size is equivalent to determine the number of groups. This type of testers is frequently used in the case of sudden death testing. An acceptance sampling plan based on such groups of items is called a group acceptance sampling plan (GASP). If the GASP is used in conjunction with truncated life tests, it is called a GASP based on truncated life test assuming that the lifetime of product follows a certain probability distribution.

Studies regarding truncated life tests can be found in Epstein(1954), Gupta and Groll(1961), Gupta(1962), Fertig and Mann(1980), Kantam and Rosaiah(1998), Kantam(2001) et. al., Baklizi(2003), Wu and Tsai(2005), Rosaiah and Kantam(2005), Tsai and Wu(2006), Balakrishnan et. al.(2007), Aslam(2007), Srinivasa Rao et. al.(2008), Aslam and Kantam(2008), Aslam et. al.(2009), Srinivasa Rao et. al.(2009), Srinivasa Rao (2009), Lio et. al.(2010) , Srinivasa Rao et al(2010), Srinivasa Rao(2011), Aslam et. al.(2011), Aslam et. al.(2011a), Ramaswamy and Anburajan (2012).

In this paper, we describe the proposed group acceptance sampling plan(GASP) based on truncated life tests when the lifetime of a product follows half normal distribution in Section 2 . Operating characteristic (OC) is given in section 3. Producer's risk is given in Section 4. Examples are provided for an illustration in Section 5. The article is closed with summary and conclusions in Section 6.

2. The Group Acceptance Sampling Plans (GASP)

Statistical methods dealing with the properties and applications of the half normal distribution have been extensively used in diverse areas of applications particularly when the data are left/right truncated. The half normal distribution occurs when sampling from a normally distributed population where the signs of the negative observations are lost or are not relevant. Generally in reliability studies, the life time of a product ranges in (0,1), this study proceeds from the observation that if we have a variable which follows half normal distribution then its probability density function (pdf) is given by

$$p(x) = \frac{2\theta}{\pi} e^{-\frac{x^2\theta^2}{\pi}}, \quad x \geq 0 \tag{2.1}$$

Its cumulative distribution function (cdf) is

$$P(x) = erf\left(\frac{\theta x}{\sqrt{\pi}}\right), \quad x \geq 0 \tag{2.2}$$

Half normal distribution is an increasing failure rate (IFR) model which is most useful in reliability studies. Because of this IFR nature we are motivated to study this distribution. Assume that the life time of a product follows half normal distribution with σ as scale parameter .Its cumulative distribution function F(.) is given by

$$F(t) = erf\left(\frac{\theta \frac{t}{\sigma}}{\sqrt{\pi}}\right), \quad t \geq 0, \sigma > 0 \tag{2.3}$$

Given 0 < q < 1, the 100th percentile is given by

$$t_q = \sigma \frac{\sqrt{\pi}}{\theta} erf^{-1}(q) \tag{2.4}$$

Substituting σ in the Equation 2.3 in the scaled form we get

$$F(t) = erf\left(\frac{\theta\left(\frac{t}{t_q}\right)\frac{\sqrt{\pi}}{\theta} erf^{-1}(q)}{\sqrt{\pi}}\right), \tag{2.5}$$

$$F(t) = erf\left(\delta erf^{-1}(q)\right), \tag{2.6}$$

Where $\delta = \frac{t}{t_q}$. When the distribution is symmetric it is clear that the mean and median are same. But when the distribution is skewed which means that the side of the tail is long through which one can expect the mean to lean towards that side of the distribution. By increasing the amount of skewness, we can make the mean become much bigger and bigger in which case the proportion of the population below the mean can be made unduly large. This is what is meant by saying that mean would not represent a center of the distribution since more than 80% of the population may be below the mean.

But if one were to use the median, there is always only 50% of the population less than the median.

Since for our present skewed population the median is a more approximate average for decision making about the quality of the life than population mean, we take $q = 0.50$. Let μ be the true value of the median of the lifetime distribution of a product and μ_0 denote the specified median under the assumption that the life time of an item follows half normal distribution. Based on the failure data, we want to test the hypothesis $H_0 : \mu \geq \mu_0$ against $H_1 : \mu < \mu_0$. A lot is considered as good if $\mu \geq \mu_0$ and bad if $\mu < \mu_0$. This hypothesis is tested using the group acceptance sampling scheme as:

1. Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be $n = g.r$.
2. Select the acceptance number *c* for a group and the experiment time t_0.
3. Perform the experiment for the *g* groups simultaneously and record the number of failures for each group.
4. Accept the lot if at most *c* failures occur in each of all groups.
5. Terminate the experiment if more than c failures occur in any group and reject the lot.

We are interested in determining the number of groups g required for in the case of the half logistic distribution and various values of acceptance number c, whereas the group size r and the termination time t_0 are assumed to be given. Since it is convenient to set the termination time as a multiple of the specified value μ_0 of the median, we will consider $t_0 = \delta\mu_0$ for a given constant (termination ratio). The probability (α) of rejecting a good lot is called the producer's risk, whereas the probability (β) of accepting a bad lot is known as the consumer's risk. The parameter value g of the proposed sampling plan is determined for ensuring the consumer's risk β. Often, the consumer's risk β is expressed by the consumer's confidence level. If the confidence level is p^*, then the consumer's risk will be $\beta = 1 - p^*$. We will determine the number of groups g in the proposed sampling plan so that the consumer's risk does not exceed a given value β. If the lot size is large enough, we can use the binomial distribution to develop the GASP.

According to GASP the lot of products is accepted only if there are at most c failures observed in each of the g groups. So, the lot acceptance probability is given by

$$\left(\sum_{i=0}^{c} \binom{r}{i} p_0^i \left(1-p_0\right)^{r-i} \right)^g \leq \beta \qquad (2.7)$$

Where $p_0 = F_t\left(\delta_0\right)$ is the probability of a failure during the time $t = \delta t_q^0$. To save space, only the results of small sample sizes for $\beta = 0.25, 0.10, 0.05, 0.01;\quad r = 2(1)7;\quad c = 0(1)5;$ $\delta = 0.7, 0.8, 1.0, 1.2, 1.5, 2.0$ are displayed in Table 1.

Table 1. Minimum number of groups (g) for the proposed plan in the case of half normal distribution

β	r	c	δ					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	2	2	1	1	1	1
0.25	3	1	4	4	2	2	1	1
0.25	4	2	10	7	4	3	2	1
0.25	5	3	22	14	7	4	3	2
0.25	6	4	52	30	12	7	3	2
0.25	7	5	125	64	22	10	4	2
0.10	4	0	2	2	1	1	1	1
0.10	5	1	3	3	2	2	1	1
0.10	6	2	5	4	3	2	1	1
0.10	7	3	10	7	4	3	2	1
0.10	8	4	18	11	6	3	2	1
0.10	9	5	35	20	8	5	2	1
0.05	5	0	2	2	1	1	1	1
0.05	6	1	3	2	2	2	1	1
0.05	7	2	5	4	3	2	1	1
0.05	8	3	8	6	3	2	2	1
0.05	9	4	14	9	5	3	2	1
0.05	10	5	26	15	7	4	2	1
0.01	7	0	2	2	1	1	1	1
0.01	8	1	3	2	2	2	1	1
0.01	9	2	4	3	2	2	1	1
0.01	10	3	7	5	3	2	2	1
0.01	11	4	11	7	4	3	2	1
0.01	12	5	17	11	5	3	2	1

3. Operating characteristic of the sampling plan

The probability of acceptance can be regarded as a function of the deviation of the specified value μ_0 of the median from its true value μ. This function is called operating characteristic (OC) function of the sampling plan. Once the minimum number of groups g is obtained, one may be interested to find the probability of acceptance of a lot when the quality is considered to be good if $\mu \geq \mu_0$ or $\frac{\mu}{\mu_0}$. The OC is given by

$$L(P) = \left(\sum_{i=0}^{c} \binom{r}{i} p_0^i (1-p_0)^{r-i} \right)^g \tag{3.1}$$

Using Equation 3.1 the OC values can be obtained for any sampling plan. To save space we present the OC values for sampling plans with

$\frac{\mu}{\mu_0} = 2,4,6,8,10,12;\ \beta = 0.25, 0.10, 0.05, 0.01;$

$\delta = 0.7, 0.8, 1.0, 1.2, 1.5, 2.0;\ \ r = 4,6,7,9;$ are given in table 2.

Table 2. Operating characteristic values of the group sampling plan for half normal distribution

β	r	g	δ	$\frac{\mu}{\mu_0}$					
				2	4	6	8	10	12
0.25	4	9	0.7	0.8114	0.9725	0.9916	0.9964	0.9981	0.9989
0.25	4	7	0.8	0.8070	0.9715	0.9912	0.9962	0.9981	0.9989
0.25	4	4	1.0	0.8118	0.9720	0.9913	0.9963	0.9981	0.9989
0.25	4	3	1.2	0.7866	0.9675	0.9899	0.9956	0.9977	0.9987
0.25	4	2	1.5	0.7727	0.9648	0.9890	0.9952	0.9975	0.9986
0.25	4	1	2.0	0.8075	0.9706	0.9908	0.9960	0.9979	0.9988
0.10	6	5	0.7	0.6680	0.9408	0.9809	0.9916	0.9956	0.9974
0.10	6	4	0.8	0.6322	0.9317	0.9776	0.9901	0.9948	0.9969
0.10	6	3	1.0	0.5730	0.9151	0.9716	0.9874	0.9933	0.9961
0.10	6	2	1.2	0.5794	0.9147	0.9711	0.9870	0.9931	0.9959
0.10	6	1	1.5	0.6589	0.9324	0.9769	0.9896	0.9944	0.9967
0.10	6	1	2.0	0.5209	0.8941	0.9627	0.9829	0.9908	0.9945
0.05	7	5	0.7	0.5343	0.9049	0.9682	0.9858	0.9925	0.9956
0.05	7	4	0.8	0.4951	0.8913	0.9630	0.9834	0.9912	0.9948
0.05	7	3	1.0	0.4328	0.8674	0.9537	0.9790	0.9888	0.9933
0.05	7	2	1.2	0.4460	0.8682	0.9533	0.9786	0.9885	0.9931
0.05	7	1	1.5	0.5457	0.8965	0.9630	0.9829	0.9908	0.9945
0.05	7	1	2.0	0.3955	0.8421	0.9413	0.9723	0.9849	0.9909
0.01	9	4	0.7	0.3810	0.8445	0.9447	0.9747	0.9864	0.9919
0.01	9	3	0.8	0.3705	0.8356	0.9405	0.9735	0.9852	0.9911
0.01	9	2	1.0	0.3608	0.8247	0.9348	0.9694	0.9833	0.9900
0.01	9	2	1.2	0.2377	0.7549	0.9045	0.9543	0.9748	0.9847
0.01	9	1	1.5	0.3510	0.8089	0.9254	0.9640	0.9800	0.9878
0.01	9	1	2.0	0.2111	0.7232	0.8856	0.9432	0.9680	0.9800

4. Producer's Risk

The producer may be interested in enhancing the quality level of the product so that the acceptance probability should be larger than a specified level. For a given value of the producer's risk, say γ , the minimum ratio can be obtained by satisfying the following inequality

$$\left(\sum_{i=0}^{c}\binom{r}{i}p_0^i\left(1-p_0\right)^{r-i}\right)^g \leq 1-\gamma \qquad (4.1)$$

To save space, the minimum values of the ratio $\frac{\mu}{\mu_0}=2$ in case of half normal distribution based on the values given in table 1 for the acceptability of a lot at the producer's risk of $\gamma = 0.05$ are presented in table 3.

Table 3. Minimum ratio of the values of true median and the specified median for the producer's risk of $\gamma = 0.05$ in the case of half normal distribution.

β	r	c	δ					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	29.5580	33.7805	21.2456	25.4947	31.8686	42.4911
0.25	3	1	5.6427	6.4488	5.6548	6.7857	5.9340	7.9120
0.25	4	2	3.3588	3.3929	3.4892	3.7843	4.0972	4.2572
0.25	5	3	2.4671	2.5037	2.6031	2.6860	2.7746	3.0410
0.25	6	4	2.0645	2.0999	2.1554	2.2984	2.3759	2.8857
0.25	7	5	1.8306	1.8577	1.9147	1.9857	2.0841	2.4226
0.10	4	0	58.9289	67.3473	42.2257	50.6708	63.3385	84.4513
0.10	5	1	8.7541	10.0046	10.1248	12.1498	10.5419	14.0558
0.10	6	2	4.4043	4.6469	5.2363	5.4204	5.2368	6.9824
0.10	7	3	3.1683	3.2841	3.5145	3.8890	4.3296	4.7113
0.10	8	4	2.4976	2.5583	2.7880	2.8486	3.2333	3.6368
0.10	9	5	2.1402	2.2031	2.3112	2.5290	2.6264	3.0789
0.05	5	0	73.6142	84.1305	52.7154	63.2584	79.0730	105.4307
0.05	6	1	10.6817	9.8752	12.3439	14.8127	12.8278	17.1037
0.05	7	2	5.2734	5.5610	6.2619	6.4748	6.2408	8.3211
0.05	8	3	3.5151	3.7091	3.8127	4.0701	5.0876	5.5234
0.05	9	4	2.7519	2.8463	3.1080	3.3071	3.7496	4.2088
0.05	10	5	2.3355	2.4040	2.5909	2.7798	3.0112	3.4573
0.01	7	0	102.9848	117.6969	73.6945	88.4334	110.5418	147.3890
0.01	8	1	14.5283	13.4173	16.7717	20.1260	17.3878	23.1837
0.01	9	2	6.4571	6.6414	7.1427	8.5713	8.2360	10.9813
0.01	10	3	4.4068	4.5805	4.9456	5.2710	6.5887	7.1300
0.01	11	4	3.3337	3.4326	3.7609	4.2118	4.7677	5.3350
0.01	12	5	2.7161	2.8513	3.0439	3.2870	3.7703	4.3164

5. Tables and Examples

The design parameters of GASP are found at various values of the consumer's risk and the test termination time multiplier in Table 1. It should be noted that if one needs the minimum sample size, it can be obtained by n = r x g. Table 1 indicates that, as the test termination time multiplier δ increases, the number of groups g decreases, i.e., a smaller number of groups is needed, if the test termination time multiplier increases at a fixed group size. For an example, from Table 1, if $\beta = 0.01$, $r = 6$, $c = 2$ and a change from 0.7 to 0.8, the required values of the design parameters of GASP change from $g = 5$ to $g = 4$. However, this trend is not monotonic since it depends on the acceptance number as well. The probability of acceptance for the lot at the median ratio corresponding to the producer's risk is given in Table 2. Finally, Table 3 presents the minimum ratios of true median to the specified median for the acceptance of a lot with producer's risk $\gamma = 0.05$ for given parameter values.

Suppose that the lifetime of a product follows the half normal distribution, it is desired to design a GASP to test if the median is greater than 1,000 hours based on a testing time of 700 hours and using testers equipped with 6 items each. It is assumed that $c = 2$ and $\beta = 0.10$. This gives the termination multiplier $\delta = 0.700$. From Table 1 the minimum number of groups required is obtained as $g = 5$. Thus, we will draw a random sample of size $n = 30$ items and allocate 6 items to each of the 5 groups to put on test for 700 hours. This indicates that a total of 30 products are needed and that 6 items are allocated to each of the 5 testers. We will accept the lot if no more than 2 failures occur before 700 hours in each of 5 groups. We terminate the experiment as soon as the 3rd failure occurs before the 700th hours. For this proposed sampling plan the probability of acceptance is $p = 0.9408$ when the true value of the median is $\mu = 4,000$ hours. This shows that, if the true value of the median is 4 times of the required value $\mu_0 = 1000$ hours, the producer's risk is $\alpha = 0.0592$. If we need the ratio to assure a producer's risk of $\gamma = 0.05$, we can obtain it from Table 3. For example, when $\beta = 0.10$, $r = 6$, $g = 5$, $c = 2$ and $\delta = 0.700$, the required ratio is $\frac{\mu}{\mu_0} = 4.4043$.

6. Summary and Conclusions

In this paper, a group acceptance sampling plan from a truncated life test is proposed in the case of half normal distribution. The number of groups and the acceptance number are determined when the consumer's risk (β) and the other plan parameters are specified. It is observed that the minimum number of groups required decreases as the test termination time multiplier increases. Moreover, the operating characteristics function increases disproportionately when the quality improves. This GASP can be used when a multiple number of items are tested simultaneously. Clearly, such a tester would be beneficial in terms of test time and test cost.

Acknowledgments

The authors thank the editor, reviewers and Prof. N. Balakrishnan for their helpful suggestions, comments and encouragement, which helped in improving the final version of the paper.

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