An Estimator of the Mean Estimation of Study Variable Using Median of Auxiliary Variable

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ABSTRACT

In the present study, I propose a modified estimator for estimating the population mean of the study variable auxiliary information when the population mean and the population median of the auxiliary variable is known. The expression of bias and mean squared error (MSE) of the proposed estimator is derived. Some existing estimators are also discussed. Comparisons of the proposed estimator with the other estimators are carried out. The results obtained are illustrated numerically by using three natural populations considered in the literature.

Keywords: Mean squared error, bias, median, simple random sampling

1. Introduction

1.1 Introduction to the Research Problem

Use of auxiliary information in the estimation of population parameters such as population mean, ratio of two population means, product of two population means, coefficient of variation etc. has been in practice. Ratio, product and regression type estimators are good examples in this context. Cochran (1940) initiated the use of auxiliary information at estimation stage and proposed ratio estimator for population mean. It is well established fact that ratio type estimators provide better efficiency in comparison to simple mean estimator if the study variable and auxiliary variable are positively correlated. If the correlation between the study variable and auxiliary variables negative, product estimator given by Robson (1957) is more efficient than simple mean estimator.

Further improvements are also achieved on the classical ratio estimator by introducing a large number of modified ratio estimators with the use of known parameters like, coefficient of variation, coefficient of kurtosis, coefficient of skewness and population correlation coefficient. For more detailed discussion one may refer to Cochran (1977), Kadilar and Cingi (2004, 2006), Koyuncu and Kadilar (2009), Murthy (1967), Prasad (1989), Rao (1991), Singh (2003), Singh and Tailor (2003, 2005), Singh et al (2004), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999) and Yan and Tian (2010).

Further, Subramani and Kumarapandiyan (2013) had taken initiative by proposed modified ratio estimator for estimating the population mean of the study variable by using the population median of the auxiliary variable.

The objective of the paper is to proposed modified estimator for estimating the population mean by using the population median of the auxiliary variable.

2. Notations Used

The following are the notations used in the paper:

N: Size of the finite population; n: Sample size taken from the population of size N; y: Study variable; x: Auxiliary variable; y_i: Value of the i^{th} unit of the sample for the study variable y; x_i: Value of the i^{th} unit of the sample for the auxiliary variable x; $\bar{Y} = \sum_{i=1}^{N} y_i/N$: Population mean for study variable y for the entire population; $\bar{X} = \sum_{i=1}^{N} x_i/N$: Population mean for the auxiliary variable x for the entire population; $\bar{y} = \sum_{i=1}^{n} y_i/n$: Sample mean for the study variable y for the sample of size n; $\bar{x} = \sum_{i=1}^{n} x_i/n$: Sample mean for the auxiliary variable x for the sample of size n; $S_y^2 = \sum_{i=1}^{N} (y_i - \bar{Y})^2/(N-1)$: Population mean square of the study variable y for the entire population; $S_x^2 = \sum_{i=1}^{N} (x_i - \bar{X})^2/(N-1)$: Population mean square of the auxiliary variable x for the entire population; $S_{xy} = \sum_{i=1}^{N} (y_i - \bar{Y})(x_i - \bar{X})/(N-1)$: Population covariance between y and x for the entire population; $\rho_{yx} = \frac{S_{xy}}{S_x S_y}$: Correlation coefficient between y and x in the entire population; $C_y = S_y/\bar{Y}$: Coefficient of variation (CV) of the study variable y in the entire population; $C_x = S_x/\bar{X}$: Coefficient of variation (CV) of the auxiliary variable x in the entire population; , $\beta_1 = \frac{N\sum_{i=1}^{N}(X_i-\bar{X})^3}{(N-1)(N-2)S^3}$ is the coefficient of Skewness of the auxiliary variable, $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i-\bar{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$ is the coefficient of Kurtosis of the auxiliary variable, and M_d is the population median of the auxiliary variable.

3. Procedure and Definitions

Consider a finite population $U = (U_1, U_2, \dots, U_N)$ of size N. Let y and x denote the study variable and the auxiliary variable taking values y_i and x_i respectively on the i^{th} unit (i = 1, 2,…,N). For estimating the population mean $\bar{Y}$ of y a simple random sample of size n is draw without replacement from the population U. Then the classical ratio estimator is defined as

$t_R = \frac{\bar{y}}{\bar{x}}\bar{X}$; if $\bar{x} \neq 0$

and the product estimator is given by

$t_P = \frac{\bar{y}}{\bar{X}}\bar{x}$;

where $\bar{X}$, the population mean of the auxiliary variable x is known.

The mean squared error expressions of the ratio and product estimators are

$$MSE(t_R) = \left(\frac{1-f}{n}\right)\bar{Y}^2\left\{C_y^2 + C_x^2\left(1 - 2\rho_{yx}\frac{C_y}{C_x}\right)\right\}$$

$$MSE(t_P) = \left(\frac{1-f}{n}\right)\bar{Y}^2\left\{C_y^2 + C_x^2\left(1 + 2\rho_{yx}\frac{C_y}{C_x}\right)\right\}$$

Further, a list of modified ratio estimators is given in table 1 is used for assessing the performance of the proposed estimator along with their bias and mean squared error expressions.

Table 1: Existing modified ratio type estimators with their biases and mean squared errors

Estimator	Bias, B(.)	Mean squared error, MSE(.)	Constants θ_i or R_i
$t_1 = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$ Sisodia and Dwivedi (1981)	$\left(\frac{1-f}{n}\right)\bar{Y}(\theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho_{yx})$	$\left(\frac{1-f}{n}\right)\bar{Y}^2(C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho_{yx})$	$\theta_1 = \frac{\bar{X}}{\bar{X}+C_x}$
$t_2 = \bar{y}\left(\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right)$ Singh et al (2004)	$\left(\frac{1-f}{n}\right)\bar{Y}(\theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho_{yx})$	$\left(\frac{1-f}{n}\right)\bar{Y}^2(C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho_{yx})$	$\theta_2 = \frac{\bar{X}}{\bar{X}+\beta_2}$
$t_3 = \bar{y}\left(\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right)$ Yan and Tian (2010)	$\left(\frac{1-f}{n}\right)\bar{Y}(\theta_3^2 C_x^2 - 2\theta_3 C_x C_y \rho_{yx})$	$\left(\frac{1-f}{n}\right)\bar{Y}^2(C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_x C_y \rho_{yx})$	$\theta_3 = \frac{\bar{X}}{\bar{X}+\beta_1}$
$t_4 = \bar{y}\left(\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}}\right)$ Singh and Tailor (2003)	$\left(\frac{1-f}{n}\right)\bar{Y}(\theta_4^2 C_x^2 - 2\theta_4 C_x C_y \rho_{yx})$	$\left(\frac{1-f}{n}\right)\bar{Y}^2(C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_x C_y \rho_{yx})$	$\theta_4 = \frac{\bar{X}}{\bar{X}+\rho_{yx}}$
$t_5 = \bar{y}\left(\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2}\right)$ Upadhyaya and Singh (1999)	$\left(\frac{1-f}{n}\right)\bar{Y}(\theta_5^2 C_x^2 - 2\theta_5 C_x C_y \rho_{yx})$	$\left(\frac{1-f}{n}\right)\bar{Y}^2(C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_x C_y \rho_{yx})$	$\theta_5 = \frac{\bar{X}C_x}{\bar{X}C_x+\rho_{yx}}$
$t_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}\bar{X}$ Kadilar and Cingi (2004)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_6^2$	$\left(\frac{1-f}{n}\right)\left(R_6^2 S_x^2 + S_y^2(1-\rho_{yx}^2)\right)$	$R_6 = \frac{\bar{Y}}{\bar{X}}$

Table 1 (continued):

$t_7=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}+C_x}(\bar{X}+C_x)$ Kadilar and Cingi (2004)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_7^2$	$\left(\frac{1-f}{n}\right)\left(R_7^2S_x^2+S_y^2\left(1-\rho_{yx}^2\right)\right)$	$R_7=\frac{\bar{Y}}{\bar{X}+C_x}$
$t_8=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}+\beta_2}(\bar{X}+\beta_2)$ Kadilar and Cingi (2004)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_8^2$	$\left(\frac{1-f}{n}\right)\left(R_8^2S_x^2+S_y^2\left(1-\rho_{yx}^2\right)\right)$	$R_8=\frac{\bar{Y}}{\bar{X}+\beta_2}$
$t_9=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}C_x+\beta_2}(\bar{X}C_x+\beta_2)$ Kadilar and Cingi (2004)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_9^2$	$\left(\frac{1-f}{n}\right)\left(R_9^2S_x^2+S_y^2\left(1-\rho_{yx}^2\right)\right)$	$R_9=\frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}$
$t_{10}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}+\beta_1}(\bar{X}+\beta_1)$ Yan and Tian (2010)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_{10}^2$	$\left(\frac{1-f}{n}\right)\left(R_{10}^2S_x^2+S_y^2\left(1-\rho_{yx}^2\right)\right)$	$R_{10}=\frac{\bar{Y}}{\bar{X}+\beta_1}$
$t_{11}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}\beta_1+\beta_2}(\beta_1\bar{X}+\beta_2)$ Yan and Tian (2010)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_{11}^2$	$\left(\frac{1-f}{n}\right)\left(R_{11}^2S_x^2+S_y^2\left(1-\rho_{yx}^2\right)\right)$	$R_{11}=\frac{\bar{Y}\beta_1}{\bar{X}\beta_1+\beta_2}$
$t_{12}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}+\rho_{yx}}(\bar{X}+\rho_{yx})$ Kadilar and Cingi (2006)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_{12}^2$	$\left(\frac{1-f}{n}\right)\left(R_{12}^2S_x^2+S_y^2\left(1-\rho_{yx}^2\right)\right)$	$R_{12}=\frac{\bar{Y}}{\bar{X}+\rho_{yx}}$
$t_{13}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}C_x+\rho_{yx}}(\bar{X}C_x+\rho_{yx})$ Kadilar and Cingi (2006)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_{13}^2$	$\left(\frac{1-f}{n}\right)\left(R_{13}^2S_x^2+S_y^2\left(1-\rho_{yx}^2\right)\right)$	$R_{13}=\frac{\bar{Y}C_x}{\bar{X}C_x+\rho_{yx}}$
$t_{14}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}\rho_{yx}+C_x}(\bar{X}\rho_{yx}+C_x)$ Kadilar and Cingi (2006)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_{14}^2$	$\left(\frac{1-f}{n}\right)\left(R_{14}^2S_x^2+S_y^2\left(1-\rho_{yx}^2\right)\right)$	$R_{14}=\frac{\bar{Y}\rho_{yx}}{\bar{X}\rho_{yx}+C_x}$
$t_{15}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}\rho_{yx}+\beta_2}(\bar{X}\rho_{yx}+\beta_2)$ Kadilar and Cingi (2006)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_{15}^2$	$\left(\frac{1-f}{n}\right)\left(R_{15}^2S_x^2+S_y^2\left(1-\rho_{yx}^2\right)\right)$	$R_{15}=\frac{\bar{Y}\rho_{yx}}{\bar{X}\rho_{yx}+\beta_2}$
$t_{16}=\bar{y}\left(\frac{\bar{X}+M_d}{\bar{x}+M_d}\right)$ Subramani and Kumarapandiyan (2013)	$\left(\frac{1-f}{n}\right)\bar{Y}\left(R_{16}^2C_x^2-R_{16}\rho_{yx}C_xC_y\right)$	$\left(\frac{1-f}{n}\right)\bar{Y}^2\left(C_y^2+R_{16}^2C_x^2-2R_{16}\rho_{yx}C_xC_y\right)$	$R_{16}=\frac{\bar{X}}{\bar{X}+M_d}$

4. Proposed Estimator

Following Subramani and Kumarapandiyan (2013), I have proposed an estimator for estimating the population mean when the population mean and population median of the auxiliary variable is known

$$t_s = \bar{y}\left\{\alpha\left(\frac{\bar{X}+M_d}{\bar{x}+M_d}\right) + (1-\alpha)\left(\frac{\bar{x}+M_d}{\bar{X}+M_d}\right)\right\}$$

where M_d is the population median of the auxiliary variable X.
To the first degree of approximation, I have obtained the expression of bias and mean squared error (MSE) of the proposed estimator as

$$B(t_s) = \left(\frac{1-f}{n}\right)\bar{Y}\{(1-2\alpha)\emptyset\rho_{yx}C_yC_x + \alpha\emptyset^2C_x^2\}$$

and

$$MSE(t_s) = \left(\frac{1-f}{n}\right)\bar{Y}\{C_y^2 + (1-2\alpha)^2\emptyset^2C_x^2 + 2(1-2\alpha)\emptyset\rho_{yx}C_yC_x\}$$

where $\emptyset = \frac{\bar{X}}{\bar{X}+M_d}$.
The $MSE(t_s)$ will be minimum when

$$\alpha = \frac{1}{2}\left(1-\frac{\rho_{yx}C_y}{\emptyset C_x}\right) = \alpha_0(say)$$

By Substituting the minimum value of α in the proposed estimator, one can get the asymptotically optimum estimator (AOE) as

$$t_s(opt) = \bar{y}\left\{\alpha_0\left(\frac{\bar{X}+M_d}{\bar{x}+M_d}\right) + (1-\alpha_0)\left(\frac{\bar{x}+M_d}{\bar{X}+M_d}\right)\right\}$$

Thus, the optimum MSE of t_s is

$$Min.MSE(t_s) = \left(\frac{1-f}{n}\right)\bar{Y}\left(1-\rho_{yx}^2\right)C_y^2.$$

5. Efficiency Comparison

For comparison of proposed estimator with the existing estimators, I have derived the conditions for which the proposed estimator is more efficient than the existing modified ratio estimators as

$Min.MSE(t_s) \leq MSE(t_i; i = 1,2,3,4,5)$ if $\rho_{yx} \leq \frac{\theta_i C_x}{C_y}$.

$Min.MSE(t_s) \leq MSE(t_i; i = 6\ to\ 15)$ if $R_i^2 S_x^2 \geq 0$.

$Min.MSE(t_s) \leq MSE(t_{16})$ if $\rho_{yx} \leq \frac{R_{16}C_x}{C_y}$.

$Min.MSE(t_s) \leq MSE(t_R)$ if $\rho_{yx} \leq \frac{C_x}{C_y}$.

From the above conditions, it is noted that the proposed estimator is more efficient among other discussed estimators if the above conditions holds true.

6. Empirical Study

To demonstrate the performance of the suggested estimator empirically in comparison to other estimators. I have used three natural population data sets. The descriptions of the populations are given below.

Population I: [Source: Murthy (1967)]

Y: Output for 80 factories in a region

X: Fixed Capital

$N = 80; n = 20;\ \bar{Y} = 51.8264;\ \bar{X} = 11.2646;\ \rho_{yx} = 0.9413;\ S_y = 18.3569;\ C_y = 0.3542;\ S_x = 8.4563;\ C_x = 0.7507;\ \beta_1 = 1.05;\ \beta_2 = -0.06339;\ M_d = 7.575$

Population II: [Source: Murthy (1967)]

Y: Output for 80 factories in a region

X: Data on number of workers

$N = 80; n = 20;\ \bar{Y} = 51.8264;\ \bar{X} = 2.8513;\ \rho_{yx} = 0.9150;\ S_y = 18.3569;\ C_y = 0.3542;\ S_x = 2.7042;\ C_x = 0.9484;\ \beta_1 = 0.6978;\ \beta_2 = 1.3005;\ M_d = 1.48$

Population III: [Source: Cochran (1977)]

Y: Number of persons

X: Number of rooms

$N = 10; n = 4;\ \bar{Y} = 101.00;\ \bar{X} = 58.80;\ \rho_{yx} = 0.6515;\ S_y = 14.6523;\ C_y = 0.1449;\ S_x = 7.5339;\ C_x = 0.1281;\ \beta_1 = 0.5764;\ \beta_2 = -0.3814;\ M_d = 58$

Here, I have computed mean squared error (MSE) and the constants of the estimators.

The results are given in the following table.

Table 2: The mean squared errors and constants of the existing and proposed estimators

Estimator	MSE and Constants					
	Population I		Population II		Population III	
	MSE	Constants	MSE	Constants	MSE	Constants
t_1; Sisodia and Dwivedi(1981)	15.25812	0.937521	17.18812	0.750401	20.2396	0.997826
t_2;Singh et al (2004)	19.33825	1.005659	12.84257	0.686762	20.14213	0.990292
t_3;Yan and Tian (2010)	14.01128	0.914735	21.36603	0.803387	20.35575	1.006529
t_4;Singh and Tailor (2003)	14.45027	0.922882	17.68491	0.757056	20.12623	0.989041
t_5;Upadhyaya and Singh (1999)	19.45917	1.007553	12.1351	0.675254	19.45448	0.928916

Table 2 (continued):

t_6;Kadilar and Cingi (2004)	58.20263	4.60082	92.65628	18.17641	43.70438	1.719388
t_7; Kadilar and Cingi (2004)	51.3313	4.313367	53.07362	13.6396	43.59507	1.71565
t_8; Kadilar and Cingi (2004)	58.84691	4.626857	44.78744	12.48287	43.21808	1.702697
t_9; Kadilar and Cingi (2004)	4.130342	1.001632	2.317192	0.974223	26.33849	0.95739
t_{10};Yan and Tian (2010)	48.93561	4.208533	60.5325	14.60269	44.0341	1.730613
t_{11}; Yan and Tian (2010)	58.81598	4.625611	35.18871	10.99178	45.04985	1.764745
t_{12}; Kadilar and Cingi (2006)	49.78532	4.246012	53.98248	13.76056	43.15575	1.700546
t_{13}; Kadilar and Cingi (2006)	47.40103	4.139986	52.63652	13.58105	39.85643	1.58251
t_{14}; Kadilar and Cingi (2006)	50.94479	4.296627	50.78761	13.33051	43.53689	1.713657
t_{15}; Kadilar and Cingi (2006)	58.88745	4.628491	42.40512	12.12991	42.96371	1.693901
t_{16};Subramani and Kumarapandiyan (2013)	2.782544	0.597921	11.13659	0.658301	19.89921	0.503425
t_s;(Proposed Estimator)	**0.068028**	**0.597921**	**0.006226**	**0.658301**	**6.267063**	**0.503425**

From the above table, it is envisaged that the proposed optimum estimator is more efficient than other existing estimator mentioned in Table 1 in terms of less mean squared error (MSE).

7. Conclusion

In this paper, I have proposed a modified estimator based on simple random sampling without replacement by using auxiliary variable, under the situation when population mean and median of the auxiliary variable is known. It is found that the performance of the proposed estimator in terms of bias and mean squared error is more efficient than all other existing estimator for certain known population parameters of auxiliary variable. The above results are supported theoretically and empirically by three natural populations.

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