Modeling and Forecasting Mortality Using the Lee-Carter Model for Indian Population Based on Decade-wise Data

Rajendra N. Chavhan and Ramkrishna L. Shinde*

Department of Statistics and Actuarial Science, School of Mathematical Sciences, North Maharashtra University, Jalgaon Jalgaon-425001, Maharashtra, India

*Corresponding Author: rlshinde@nmu.ac.in

Received: 25th February 2015 / Revised: 29th February 2016 / Accepted: 14th March 2016

ABSTRACT

The stochastic mortality model given by Lee and Carter (1992) has been used in literature for fitting and forecasting the human mortality. We have modeled mortality rates of Indian population using the Lee-Carter (LC) model based on decade-wise data available separately for Indian female and male populations in the form of life tables for the period 1901-2011. The Singular Value Decomposition (SVD) approach is used for estimation of the parameters of the LC model. Forecasted values of time dependent parameter k_t *of the LC model are obtained for next five decades using best fitted auto regressive integrated moving average (ARIMA) model. Forecasted values of life expectancy at different ages with 95% confidence intervals are also reported for the next five decades. As an application, using forecasted mortality rates for the next five decades, net single premium for whole life and term insurance, actuarial present values of life annuities for some selected ages are also evaluated.*

Keywords: Lee-Carter (LC) model, Mortality modeling, Forecasting, Life expectancy, Singular value decomposition (SVD), Net single premium (NSP).

1. Introduction

Mortality modeling and its forecasting have been a major research area for the practitioners in actuarial science and demography. During the last century many countries have experienced considerable improvements in mortality rates. Dramatic decline in mortality brings very serious financial exposures for insurers providing life contracts, life annuities. Population forecasts are used for policy making and pricing insurance products.

Lee and Carter (1992) introduced the first mortality model with stochastic forecast. In this model time dependent factor k_t is modeled through time series models. Singular Value Decomposition (SVD) was used by Lee and Carter (1992) for the parameter estimation. Due to its simplicity in the parameter estimation, Lee-Carter (LC) model became more popular. The LC model has been used for fitting and forecasting the mortality rates for many countries: US (Lee and carter, 1992), Chile (Lee and Rofman, 1994), China (Lin,1995), Japan (Wilmoth, 1996), the seven most economically developed nations (G7) (Tuljapurkar et al., 2000), India (Singh and Ram,2004 and Yadav et. al. 2012), the Nordic countries (Kossi et. al., 2006), Sri Lanka (Aberathna et. al., 2014) and Thai (Yasungnoen and Sattayatham, 2016). Singh and Ram (2004) modeled Indian mortality by LC model and forecasted life expectancy at birth using Sample Registration System (SRS) data for the years 1970 to 2002. Yadav et.al (2012) also used LC model and forecasted age specific death rates up to year 2025 for Indian female and male population based on SRS data for the years 1981 to 2006. In the present work, we have used the decade-wise life table data (from 1901-11 to 2001-11) and modeled the Indian mortality by LC model using SVD. We have forecasted the future mortality rates for both female and male populations for the period decades 1911-21 to 1951-61. Forecasted values of life expectancy at different ages with 95% confidence interval are obtained and reported for next five decades. Actuarial quantities evaluated based on forecasted mortality are also reported.

The rest of the paper is organized as follows: In Section 2, we discuss the data used in this paper. In Section 3, we have presented some results about changes in mortality pattern in India during last century. The LC model and modeling procedure is discussed in Section 4. Section 5 gives the results of fitted model. Section 6 deals with results on the mortality forecasting for India. In Section 7, as an application of study, we have presented some actuarial calculation. Conclusions are reported in Section 8.

2. Data Description

Our study is based on data obtained from abridged life tables of Indian population for 11 decades from 1901-1911 to 2001-2011. These life tables are taken from the websites www.lifetable.de and www.who.org.in. Life tables of Indian female and male populations are separately available for the decades 1901-1911 to 1971-1981 for the ages 0, 1-4 and 5-80 (quinquennial) and 80+ whereas for the decades 1981-1991 to 2001-2011 these life tables are available for the ages 0, 1-4 and 5-100 (quinquennial) and 100+. To maintain uniformity in our data, we have converted the life tables of the decades 1981-91 to 2001-11 for the ages 0, 1-4 and 5-80 (quinquennial) and 80+. From these life tables, we have derived decade-wise age group specific central death rates ($m_{x,t}$) as

$$m_{x,t} = \frac{D_{x,t}}{L_{x,t}} \tag{1}$$

where,
$D_{x,t}$ denotes the number of deaths in the age group x during decade t.
$L_{x,t}$ denotes the average number of persons in the age group x during decade t.

$m_{x,t}$ is also known as central rate of mortality or central mortality rate for age group x during decade t. Data on decade-wise age group specific central death rates for Indian female and male populations are reported in Table 1 and Table 2 respectively.

Table 1: Decade-wise age group specific central death rates for Indian female population during 1901-1911 to 2001-2011

Age Group	**1901-11**	**1911-21**	**1921-31**	**1931-41**	**1941-51**	**1951-61**	**1961-71**	**1971-81**	**1981-91**	**1991-01**	**2001-11**
0	0.3731	0.3789	0.2991	0.2549	0.2154	0.1664	0.1556	0.1449	0.0899	0.0703	0.0510
1-4	0.0574	0.0583	0.0463	0.0394	0.0331	0.0250	0.0232	0.0213	0.0096	0.0066	0.0039
5-9	0.0143	0.0139	0.0129	0.0114	0.0098	0.0078	0.0060	0.0044	0.0034	0.0023	0.0012
10-14	0.0114	0.0111	0.0103	0.0091	0.0078	0.0062	0.0048	0.0035	0.0017	0.0014	0.0009
15-19	0.0176	0.0171	0.0159	0.0141	0.0120	0.0096	0.0073	0.0054	0.0029	0.0024	0.0014
20-24	0.0227	0.0220	0.0204	0.0181	0.0155	0.0123	0.0095	0.0069	0.0037	0.0031	0.0019
25-29	0.0233	0.0227	0.0210	0.0186	0.0160	0.0127	0.0097	0.0071	0.0034	0.0032	0.0018
30-34	0.0243	0.0236	0.0219	0.0194	0.0166	0.0132	0.0101	0.0074	0.0033	0.0031	0.0020
35-39	0.0268	0.0260	0.0241	0.0214	0.0184	0.0146	0.0112	0.0082	0.0039	0.0033	0.0025
40-44	0.0308	0.0299	0.0277	0.0246	0.0211	0.0168	0.0129	0.0094	0.0048	0.0041	0.0031
45-49	0.0372	0.0361	0.0335	0.0297	0.0255	0.0203	0.0157	0.0115	0.0068	0.0058	0.0042
50-54	0.0463	0.0449	0.0418	0.0371	0.0319	0.0255	0.0197	0.0144	0.0104	0.0092	0.0065
55-59	0.0594	0.0576	0.0536	0.0478	0.0412	0.0342	0.0256	0.0188	0.0162	0.0152	0.0109
60-64	0.0776	0.0753	0.0702	0.0628	0.0544	0.0439	0.0342	0.0253	0.0269	0.0220	0.0201
65-69	0.1056	0.0995	0.0931	0.0838	0.0731	0.0596	0.0467	0.0348	0.0436	0.0381	0.0312
70-74	0.1296	0.1318	0.1240	0.1125	0.0992	0.0819	0.0651	0.0490	0.0649	0.0556	0.0540
75-79	0.1776	0.1734	0.1644	0.1511	0.1352	0.1138	0.0921	0.0707	0.0843	0.0823	0.0746
80+	0.3145	0.3120	0.3074	0.3004	0.2916	0.2793	0.2660	0.2522	0.1578	0.1302	0.1429

3. Mortality Pattern in India

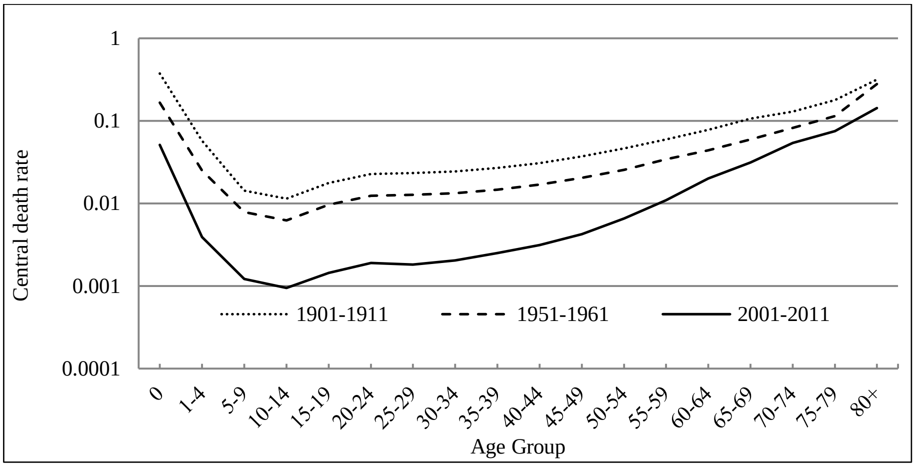
To demonstrate the improvement in Indian mortality in Figure 1, we have plotted age group specific central death rates for Indian female and male

Table 2: Decade-wise age group specific central death rates for Indian male population during 1901-1911 to 2001-2011

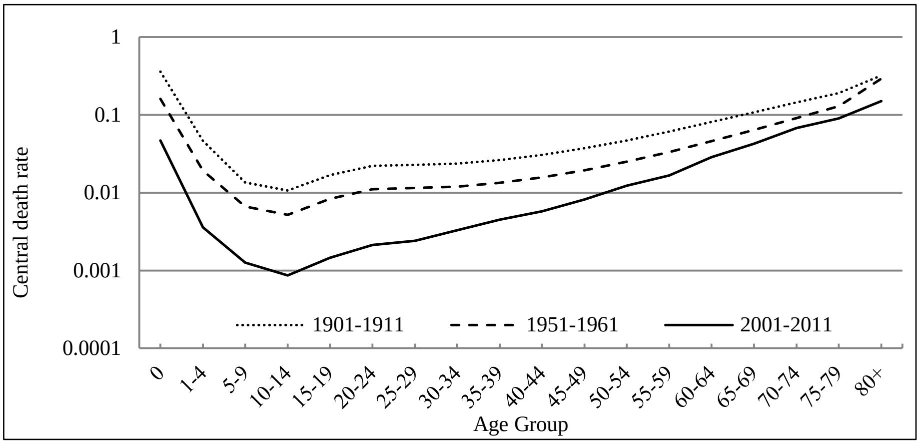
Age Group	1901-11	1911-21	1921-31	1931-41	1941-51	1951-61	1961-71	1971-81	1981-91	1991-01	2001-11
0	0.35864	0.36419	0.28804	0.24569	0.20785	0.16071	0.15031	0.14005	0.08210	0.06432	0.04672
1-4	0.04623	0.04692	0.03719	0.03152	0.02635	0.01925	0.01755	0.01593	0.00881	0.00609	0.00357
5-9	0.01355	0.01330	0.01227	0.01065	0.00869	0.00665	0.00478	0.00323	0.00280	0.00187	0.00126
10-14	0.01065	0.01043	0.00961	0.00832	0.00677	0.00517	0.00371	0.00250	0.00165	0.00141	0.00087
15-19	0.01687	0.01656	0.01531	0.01332	0.01089	0.00837	0.00603	0.00408	0.00199	0.00184	0.00145
20-24	0.02208	0.02172	0.02013	0.01757	0.01442	0.01111	0.00804	0.00545	0.00276	0.00253	0.00213
25-29	0.02275	0.02239	0.02076	0.01812	0.01487	0.01147	0.00830	0.00563	0.00307	0.00322	0.00241
30-34	0.02371	0.02335	0.02166	0.01892	0.01554	0.01199	0.00868	0.00589	0.00364	0.00385	0.00328
35-39	0.02634	0.02594	0.02409	0.02107	0.01733	0.01340	0.00972	0.00661	0.00462	0.00511	0.00448
40-44	0.03051	0.03008	0.02797	0.02452	0.02022	0.01567	0.01140	0.00777	0.00637	0.00644	0.00578
45-49	0.03725	0.03678	0.03428	0.03015	0.02495	0.01941	0.01417	0.00969	0.01038	0.00932	0.00820
50-54	0.04700	0.04648	0.04346	0.03839	0.03194	0.02499	0.01834	0.01260	0.01522	0.01374	0.01234
55-59	0.06106	0.06050	0.05663	0.05046	0.04230	0.03334	0.02465	0.01706	0.02380	0.02190	0.01669
60-64	0.08086	0.08027	0.07575	0.06781	0.05739	0.04573	0.03416	0.02387	0.03627	0.03061	0.02850
65-69	0.10798	0.10744	0.10205	0.09227	0.07912	0.06399	0.04853	0.03438	0.05407	0.04950	0.04240
70-74	0.14400	0.14352	0.13738	0.12589	0.10985	0.09063	0.07023	0.05076	0.07709	0.06631	0.06750
75-79	0.18961	0.18946	0.18296	0.17031	0.15195	0.12891	0.10293	0.07666	0.10305	0.09781	0.09022
80+	0.32154	0.32155	0.31860	0.31263	0.30334	0.29080	0.27562	0.25890	0.16049	0.13974	0.15024

populations. Overall there has been decline in mortality rates in India during 1901 to 2011 for both female and male populations. We have observed higher level of decline in mortality rates during 1961 to 2011 as compared to 1901 to 1961. Also rapid decline is observed in central mortality rates of Indian female population than male population from 1951-61 to 2001-11 for the ages 35-74 years.

Using data on life tables we have obtained life expectancies (LE) for female and male populations for all decades. Results obtained on LE are reported in Table 3 for some selected age groups. In India, Life expectancy at birth has been increased from 23.23 years to 67.30 years for female and from 24.76 years to 63.81 years for male during last century. During decades 1901-11, 1911-21 and 1921-31, life expectancy of Indian female and male at age group 20-24 was higher than life expectancy at birth. This is due to high rate of infant mortality during these decades.



(a) Female Population



(b) Male Population

Figure 1: Pattern of age group specific central death rates for Indian Population

4. Lee-Carter Model and Parameter Estimation

4.1 Lee-Carter Model

Lee and Carter (1992) introduced the first stochastic mortality model for modeling human mortality as given by,

$$ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \tag{2}$$

Table 3: Life expectancy (in years) of Indians during 1901-2011 at selected age groups

Decade	At Age Group							
	At Birth		20-24		40-44		60-64	
	Female	Male	Female	Male	Female	Male	Female	Male
1901-11	23.23	24.76	27.10	27.21	18.08	17.91	9.29	8.98
1911-21	23.34	24.74	27.55	27.42	18.39	18.02	9.47	9.02
1921-31	26.97	28.26	28.68	28.48	19.15	18.69	9.89	9.33
1931-41	30.25	31.63	30.51	30.42	20.38	19.93	10.56	9.96
1941-51	34.09	35.91	32.85	33.26	21.97	21.78	11.44	10.92
1951-61	39.89	42.04	36.24	36.96	24.23	24.27	12.77	12.27
1961-71	44.27	46.98	40.23	41.41	27.05	27.35	14.32	14.01
1971-81	49.13	52.45	44.59	46.34	30.14	30.86	16.11	16.06
1981-91	58.70	57.58	48.79	46.27	31.70	28.84	15.82	14.21
1991-01	62.59	60.46	50.60	47.32	33.29	30.07	17.21	15.33
2001-11	67.30	63.81	52.92	48.73	34.71	31.09	17.67	15.72

where,
x denotes age group under consideration (here in our data, x =0, 1-4, 5-9,…,75-79, 80+),

t denotes period (decade) of life table under consideration (here in our data, t=1901-11, 1911-21, … , 2001-11),

$m_{x,t}$ is the central rate of mortality for age group x at time t,

a_x denotes the coefficient which describes average age specific pattern of mortality,

k_t denotes the time trend for the general mortality, b_x denotes the coefficient which measures sensitivity of $ln(m_{x,t})$ at age-grouping x as the k_t varies and

$\varepsilon_{x,t}$ denotes the error associated with age-grouping x and time t.

The parameterization in (2) is invariant to transformations like $(a_x, b_x, k_t) \longrightarrow \left(a_x + cb_x, \frac{b_x}{d},\ d(k_t - c)\right)$ for some constant c and d $(d \neq 0)$. Therefore for unique solution the following constraints used by Lee and Carter (1992),

$$\sum_x b_x = 1 \text{ and } \sum_t k_t = 0 \qquad (3)$$

4.2 Parameter Estimation by Singular Value Decomposition (SVD)

Lee and Carter (1992) estimated the parameters of the LC model given in equation (1) by the SVD method. The estimated parameter vector $\hat{a}_x$ is determined as the average over time of the logarithm of the central death rates as

$$\hat{a}_x = \frac{1}{11}\sum_t ln(m_{x,t}) \text{ for } x\text{=0,1-4, 5-9 ,...75-79 ,80+} \quad (4)$$

To obtain estimated parameters $\hat{b}_x$ and $\hat{k}_t$, we applied singular value decomposition on matrix Z, where

$$\text{Z} = \ln(m_{x,t}) - \hat{a}_x \quad (5)$$

that is

$$Z = \begin{bmatrix} ln(m_{0,1901-11}) - \hat{a}_0 & ln(m_{0,1911-21}) - \hat{a}_0 & \dots & ln(m_{0,2001-11}) - \hat{a}_0 \\ ln(m_{1-4,1901-11}) - \hat{a}_{1-4} & ln(m_{1-4,1911-21}) - \hat{a}_{1-4} & \dots & ln(m_{1-4,2001-11}) - \hat{a}_{1-4} \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ ln(m_{75-79,1901-11}) - \hat{a}_{75-79} & ln(m_{75-79,1911-21}) - \hat{a}_{75-79} & \cdots & ln(m_{75-79,2001-11}) - \hat{a}_{75-79} \\ ln(m_{80+,1901-11}) - \hat{a}_{80+} & ln(m_{80+,1911-21}) - \hat{a}_{80+} & \cdots & ln(m_{80+,2001-11}) - \hat{a}_{80+} \end{bmatrix} \quad (6)$$

Applying SVD to the matrix Z, we achieve the decomposition

$$SVD(Z) = \lambda_1 P_{x,1} Q_{t,1} + \lambda_2 P_{x,2} Q_{t,2} + \cdots + \lambda_k P_{x,k} Q_{t,k} \,, \quad (7)$$

where $k = rank(Z)$, λ_i $(i = 1,2,\dots,k)$ are the singular values in increasing order with $P_{x,i}$ and $Q_{t,i}$ $(i = 1,2,\dots,k)$ as the corresponding left and right singular vectors. The approximation to the first term of SVD(Z) gives the estimates $\hat{b}_x = P_{x,1}$ and $\hat{k}_t = \lambda_1 Q_{t,1}$. The LC model is popular due to its simplicity for the parameter estimation by SVD. The proportion of variation explained by the LC model is $\lambda_1^2 / \sum_{i=1}^{k} \lambda_i^2$.

5. Fitting of the LC Model to Indian Mortality Data

This section presents the results of estimation of parameters in LC model. Estimated values of age dependent parameters a_x and b_x are reported in Table 4 and estimated values of time dependent parameter k_t is reported in Table 5 for female and male populations in India based on decade-wise life tables (1901-11 to 2001-11). For SVD analysis, we have used MATLAB program.

From SVD analysis, we found that 98.16% and 96.47% variation explained by fitted LC model for Indian female and male mortality data respectively. In Figure 2, we have plotted observed and fitted age group specific central death rates for three decades, 1901-11, 1951-61 and 2001-11. We observed that the fitted mortality rates are very close to observed (actual) mortality rates except for lower and higher ages for the decade 2001-11.

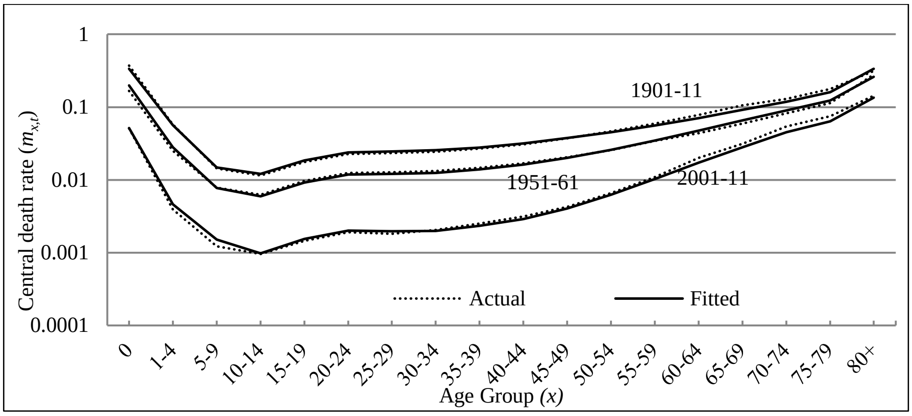
Table 4: $\hat{a}_x$ and $\hat{b}_x$ for Indian population based on decade-wise life tables (1901-11 to 2001-11)

Age Group (x)	$\hat{a}_x$		$\hat{b}_x$	
	Female	**Male**	**Female**	**Male**
0	-1.7897	-1.8410	0.0529	0.0591
1-4	-3.8089	-4.0109	0.0710	0.0721
5-9	-5.0679	-5.2037	0.0644	0.0731
10-14	-5.3599	-5.4923	0.0711	0.0781
15-19	-4.9191	-5.0586	0.0702	0.0819
20-24	-4.6638	-4.7623	0.0699	0.0793
25-29	-4.6490	-4.6972	0.0715	0.0750
30-34	-4.6194	-4.6055	0.0721	0.0687
35-39	-4.5045	-4.4505	0.0701	0.0627
40-44	-4.3463	-4.2658	0.0676	0.0587
45-49	-4.1166	-4.0037	0.0630	0.0521
50-54	-3.8347	-3.7177	0.0558	0.0466
55-59	-3.5145	-3.4045	0.0480	0.0420
60-64	-3.1804	-3.0659	0.0400	0.0366
65-69	-2.8296	-2.7170	0.0335	0.0319
70-74	-2.4944	-2.3777	0.0272	0.0286
75-79	-2.1768	-2.0521	0.0259	0.0267
80+	-1.4311	-1.3943	0.0258	0.0268

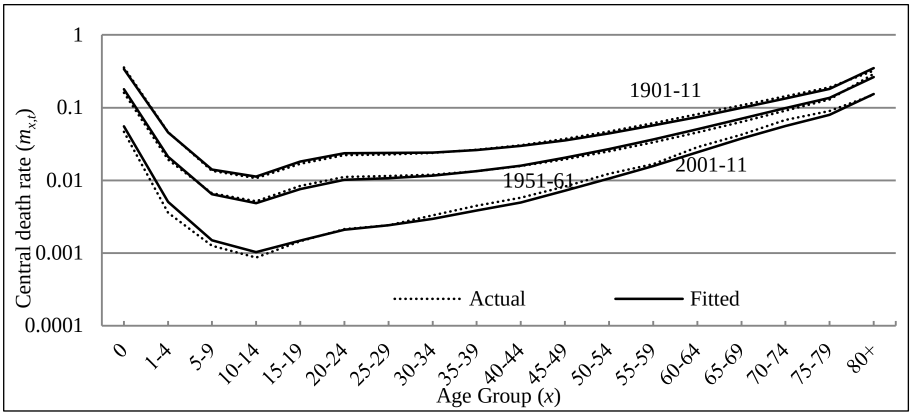
Table 5: $\hat{k}_t$ for Indian population based on decade wise life tables (1901-11 to 2001-11)

Decade	**1901-11**	**1911-21**	**1921-31**	**1931-41**	**1941-51**	**1951-61**
$\hat{k}_t$ (Female)	13.191	12.818	11.325	9.351	6.901	3.237
$\hat{k}_t$ (Male)	12.800	12.639	11.175	9.078	6.140	2.120
Decade	**1961-71**	**1971-81**	**1981-91**	**1991-01**	**2001-11**	
$\hat{k}_t$ (Female)	-0.569	-5.053	-13.069	-15.973	-22.160	
$\hat{k}_t$ (Male)	-2.328	-7.669	-12.167	-14.000	-17.788	

In Table 6, we have presented the actual LE and LE based on fitted LC model at birth and at ages 20-24, 40-44 and 60-64 for three selected decades.



(a) Female Population



(b) Male Population

Figure 2: Actual and fitted age group specific central death rates for three decades

From the values of actual and model based LE, we observed a good fit. To obtain model based LE, we have used definitions for construction of abridged life table given by Greenwood (1922) and Chiang (1984).

Table 6: Observed and fitted life expectancy at some selected age groups using the LC model

Life Expectancy at	Actual/ Fitted	Female			Male		
		1901-11	1951-61	2001-11	1901-11	1951-61	2001-11
Birth	Actual	23.23	39.89	67.30	24.76	42.04	63.81
	Fitted	24.05	38.70	66.24	25.31	41.18	63.04
20-24	Actual	27.10	36.24	52.92	27.21	36.96	48.73
	Fitted	26.89	36.35	52.11	27.25	36.68	48.97
40-44	Actual	18.08	24.23	34.71	17.91	24.27	31.09
	Fitted	18.40	23.89	33.89	18.47	23.47	31.16
60-64	Actual	9.29	12.77	17.67	8.98	12.27	15.72
	Fitted	9.90	12.00	16.56	9.41	11.55	15.22

6. Forecasting

Forecasting is the main aim behind the stochastic modeling. One of the noteworthy property of the LC model is that, once it is fitted (i.e. once values of $\hat{a}_x, \hat{b}_x$ and $\hat{k}_t$ are found), only the mortality index (k_t) over time needs to be forecasted for future time points. Lee and Carter (1992) fitted autoregressive integrated moving average (ARIMA) (0,1,0) (i.e. random walk with drift) for modeling mortality index k_t for US population and also suggested to use the appropriate ARIMA models for different populations. We have considered some possible choices of ARIMA models for modeling mortality index k_t. Table 7 includes values of Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) for considered models. According to AIC and BIC, ARIMA(1,2,0) and ARIMA(0,2,0) are best fitted models for mortality index k_t estimated by SVD for Indian female and male populations respectively. Values in Table 7 are obtained by the R 'forecast' package. (see Hyndman, and Khandakar, (2008)). We have used best fitted ARIMA models for forecasting future values of mortality index, k_t and other corresponding quantities of life table.

In the Table 8, we have given the forecasted values of mortality index, k_t along with its 95% confidence intervals (CI) for next five decades. These values are forecasted one by one for each next decade and then by adding the forecasted

Table 7: Fitted ARIMA models for estimated mortality index k_t

Model	Female		Male	
	AIC	BIC	AIC	BIC
ARIMA(2,1,2) with drift	46.36	48.18	41.69	43.51
ARIMA(2,1,1) with drift	46.37	47.88	39.71	41.22
ARIMA(1,1,2) with drift	44.74	46.25	39.74	41.26
ARIMA(1,1,1) with drift	45.78	46.99	37.82	39.03
ARIMA(1,1,0) with drift	44.57	45.48	36.04	36.95
ARIMA(0,1,1) with drift	44.97	45.87	36.04	36.94
ARIMA(0,1,0) with drift	43.33	43.93	37.44	38.04
ARIMA(1,2,0) without drift	**41.01**	**41.41**	35.03	35.42
ARIMA(0,2,0) without drift	43.50	43.70	**33.04**	**33.24**

Table 8: Forecasted values of mortality index k_t with 95% CI

Decade	Female	Male
2011-21	-26.19 (-29.78,-22.60)	-21.58 (-24.24,-18.91)
2021-31	-31.65 (-35.06,-28.24)	-25.36 (-27.89,-22.84)
2031-41	-36.16 (-39.40,-32.91)	-29.15 (-31.56,-26.74)
2041-51	-41.30 (-44.41,-38.20)	-32.94 (-35.24,-30.63)
2051-61	-46.02 (-49.00,-43.03)	-36.73 (-38.94,-34.51)

value in the previous values of k_t and again by using best fitted model, next decade's value for k_t was forecasted and so on. At each forecasting stage we got the same model with different parameters. We observed that in next five decades mortality are expected to decline for both female and male population in India. This is due to decreasing nature of k_t.

We have forecasted values of age specific death rate, m_{xt} by using estimated parameters $\hat{a}_x, \hat{b}_x$ and forecasted values of mortality index k_t. Table 9 reveals the forecasted age specific central death rates in terms of deaths per 100,000 for the five decades, 2011-21, 2021-31, 2031-41, 2041-51 and 2051-61.

We observed that infant mortality rate will decline from 42 to 15 per thousand for female and 44 to 18 for male from period 2011-21 to 2051-61. By the decade 2051-61, deaths rates for age groups between 1 to 49 years for female and age groups 5 to 34 years for male expected to be lower than one per thousand. The age specific death rates will continue to be lower for female population as compare to the male population for all five decades. Table 10 presents the LE along with 95% CI at birth and for the age groups 20-24, 40-44 and 60-64.

Table 9: Forecasted values of age specific central death rates in terms of deaths per 100,000 for Indian population

Age Group	Female					Male				
	2011-21	2021-31	2031-41	2041-51	2051-61	2011-21	2021-31	2031-41	2041-51	2051-61
0	4174	3126	2463	1875	1461	4429	3540	2830	2262	1808
1-4	345	234	170	118	84	383	291	222	169	128
5-9	117	82	61	44	33	114	86	65	49	38
10-14	73	50	36	25	18	76	57	42	31	23
15-19	116	79	58	40	29	109	80	58	43	31
20-24	151	103	75	53	38	154	114	85	63	46
25-29	147	100	72	50	36	181	136	102	77	58
30-34	149	101	73	50	36	227	175	135	104	80
35-39	177	120	88	61	44	302	238	188	148	117
40-44	220	152	112	79	58	396	317	254	203	163
45-49	313	222	167	121	90	593	487	400	328	269
50-54	501	369	287	215	166	889	745	625	524	439
55-59	847	651	525	410	327	1341	1144	976	832	709
60-64	1457	1171	978	796	659	2115	1841	1602	1395	1214
65-69	2452	2042	1756	1477	1261	3316	2938	2604	2307	2044
70-74	4049	3490	3088	2684	2361	5000	4486	4025	3611	3240
75-79	5759	5001	4451	3896	3449	7215	6520	5892	5324	4811
80+	1216	10571	9411	8242	7299	13918	12576	11363	10267	9277

LE at birth will increase from 71.1 (CI: 68.3, 73.7) to 83.6 (CI: 81.9, 85.4) for female population and from 67.2 (CI: 65.0, 69.3) to 77.5(CI: 76.2, 78.8) for male population from the decade 2011-21 to 2051-61.

By the decade 2051-61, life expectancy at age group 60-64 will reach to 26.9 years for female and 22.6 years for male population.

5. Actuarial Application

For evaluation of actuarial quantities we have used the forecasted values of central death rates in terms of number of deaths per 100,000 as reported in Table 9. We have derived and presented values of Net Single Premium (NSP) of whole life insurance, NSP for 20-year term life insurance, Actuarial Present Value (APV) of whole life annuities and 20-year temporary annuities at some selected ages. In order to find these NSP of life insurance and APV of life annuities, we need to find life table in the complete form (that is life table for all integer ages instead of groups). For the estimation of complete sets of age specific central mortality rates for each age year we have used interpolative method for initial estimates and Whittaker graduation for smoothing. This method is thoroughly discussed by Li and Chan (2004).

Table 10: Forecasted values of life expectancy at different age groups with 95% confidence intervals

Gender	Decade	Life expectancy with 95% CI at			
		Birth	20-24	40-44	60-64
Female	2011-21	71.1 (68.3,73.7)	56.1 (54.3,57.8)	37.5 (36.1,390)	20.0 (18.9,21.1)
	2021-31	75.0 (72.6,77.2)	58.7 (57.1,60.3)	39.8 (38.4,41.1)	21.7 (20.6,22.8)
	2031-41	77.8 (75.8,79.8)	60.8 (59.3,62.3)	41.6 (40.3,43.0)	23.2 (22.1,24.4)
	2041-51	80.9 (79.1,82.7)	63.2 (61.8,64.7)	43.8 (42.5,45.1)	25.1 (23.9,26.3)
	2051-61	83.6 (81.9,85.4)	65.4 (64.0,66.9)	45.9 (44.5,47.2)	26.9 (25.7,28.2)
Male	2011-21	67.2 (65.0,69.3)	52.2 (50.7,53.5)	34.0 (32.8,35.1)	17.8 (17.0,18.5)
	2021-31	70.1 (68.2,71.9)	54.1 (52.8,55.3)	35.5 (34.5,36.6)	18.9 (18.1,19.6)
	2031-41	72.7 (71.1,74.3)	56.0 (54.8,57.1)	37.1 (36.1,38.1)	20.0 (19.3,20.8)
	2041-51	75.2 (73.7,76.6)	57.8 (56.7,58.9)	38.7 (37.7,39.7)	21.3 (20.5,22.1)
	2051-61	77.5 (76.2,78.8)	59.6 (58.5,60.7)	40.3 (39.4,41.3)	22.6 (21.8,23.4)

7.1 Some Actuarial Terms and Notations:

Suppose random variable X denotes the new born's age at the time of death. That is life length of an individual, X is a non negative continuous random variable with distribution function F. In the theory of life insurance, the main random variable of interest is the time until death random variable for person aged x and it is denoted by $T(x)$. It is to be noted that distribution function of $T(x)$ is the same as conditional distribution function of $X - x$ given that $X > x$. The distribution function of $T(x)$ is given by,

$$F_{T(x)}(t) = P[(x) \le t] = P[X - x \le t \,|X > x\,] = \ {}_tq_x \qquad (8)$$

where ${}_tq_x$ denote the probability that person aged x dies in the age interval $(x, x + t\,]$.

A discrete random variable $K(x)$ associated with future life $T(x)$ is defined as the largest integer strictly smaller than $T(x)$. $K(x)$ is an integer valued random variable. It is known as curtate future lifetime at age x. The probability mass function (pmf) of $K(x)$ is obtained from distribution of $T(x)$. For $x = 0, 1, 2, \ldots$, pmf of $K(x)$ is

$$P[K(x) = k] = P[k < T(x) \leq k + 1] = {}_k p_x \times q_{x+k}, \tag{9}$$

where ${}_k p_x = 1 - {}_k q_x$.

Let i be the effective rate of interest, which is interest earned on 1 unit invested in one period (generally one period is one year) and v is the present value or discounted value of unit 1 payable after one period. Hence, $v = 1/(1 + i)$. The NSP for whole life insurance payable for one unit benefit payable at the end of year of death of person aged x years, denoted by A_x and its expression is given by,

$$A_x = \sum_{k=0}^{\infty} v^{k+1} \times {}_k p_x \times q_{x+k} \tag{10}$$

The NSP for n-year term life insurance payable for one unit benefit payable at the end of year of death of person aged x years if death occur within n year from the policy issue, denoted by $A^1_{x:\overline{n}|}$ and its expression is given by,

$$A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1} \times {}_k p_x \times q_{x+k} \tag{11}$$

The APV of whole life annuity due for one unit payment at the beginning of each year throughout the remaining lifetime of an individual now aged x denoted by $\ddot{a}_x$ and its expression is given by,

$$\ddot{a}_x = \sum_{k=0}^{\infty} v^k \times {}_k p_x \tag{12}$$

The APV of n-year temporary life annuity due for one unit payment at the beginning of each year for the next n years or till survivor whichever occur first for an individual now aged x denoted by $\ddot{a}_{x:\overline{n}|}$ and its expression is given by,

$$\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k \times {}_k p_x \tag{13}$$

Table 11 presents the values of A_x and $A^1_{x:\overline{20}|}$ calculated at 6.75% effective rate of interest per annum. (as per current repo rate on 29.02.2016 of Reserve Bank of India). In the Table 12, we have reported the APV of life annuities, $\ddot{a}_x$ and $\ddot{a}_{x:\overline{20}|}$ calculated at 6.75% effective rate of interest per annum (as per repo rate on 29.02.2016 of Reserve Bank of India). These values of NSP of life insurances or APV of life annuities may be useful for finding total cost of one time premium for person of age $x = 0, 10, 20, 40$ and 60.

Table 11: Forecasted values of NSP of whole life insurance, A_x and for 20-year term life insurance, $A^1_{x:\overline{20}|}$ at some selected ages

NSP	Gender	Decade	Age (x)				
			0	10	20	40	60
A_x	Female	2011-21	0.0673	0.0315	0.0480	0.1209	0.3226
		2021-31	0.0506	0.0249	0.0392	0.1060	0.2982
		2031-41	0.0402	0.0208	0.0336	0.0956	0.2795
		2041-51	0.0310	0.0172	0.0285	0.0854	0.2595
		2051-61	0.0246	0.0146	0.0249	0.0775	0.2425
	Male	2011-21	0.0737	0.0381	0.0608	0.1541	0.3648
		2021-31	0.0597	0.0323	0.0528	0.1405	0.3457
		2031-41	0.0485	0.0276	0.0461	0.1285	0.3274
		2041-51	0.0395	0.0238	0.0406	0.1176	0.3099
		2051-61	0.0323	0.0207	0.0359	0.1079	0.2932
$A^1_{x:\overline{20}|}$	Female	2011-21	0.0572	0.0155	0.0216	0.0551	0.2546
		2021-31	0.0423	0.0111	0.0159	0.0436	0.2257
		2031-41	0.0329	0.0086	0.0124	0.0363	0.2040
		2041-51	0.0248	0.0064	0.0095	0.0296	0.1815
		2051-61	0.0191	0.0050	0.0076	0.0248	0.1629
	Male	2011-21	0.0610	0.0170	0.0277	0.0845	0.3066
		2021-31	0.0485	0.0133	0.0223	0.0729	0.2835
		2031-41	0.0386	0.0105	0.0180	0.0629	0.2616
		2041-51	0.0307	0.0084	0.0147	0.0545	0.2409
		2051-61	0.0244	0.0068	0.0120	0.0472	0.2216

6. Conclusions

We modeled the central mortality rates of Indian population by using the LC model estimated by SVD, approach based on decade-wise mortality data from 1901-11 to 2001-11. We observed the following.

i) The general pattern of mortality ($\hat{a}_x$) for both female and male populations shown high infant mortality, an accidental hump around ages 20 years and nearly exponential increase at older ages.
ii) The sensitivity of mortality ($\hat{b}_x$) has shown mortality decline at high rate for ages 25-34 years for female and for ages 15-24 years for male population than other ages.
iii) Mortality index ($\hat{k}_t$) has shown decreasing trend.
iv) Improvement in female mortality is larger than male mortality.
v) Mortality improvement and its impact on actuarial quantities are observed at all ages for Indian female and male populations.

Table 12: Forecasted values of APV of whole life annuity, $\ddot{a}_x$ and for 20-year temporary annuity, $\ddot{a}_{x:\overline{20}|}$ at some selected ages

APV	Gender	Decade	Age (x)				
			0	**10**	**20**	**40**	**60**
$\ddot{a}_x$	Female	2011-21	14.7508	15.3161	15.0560	13.9030	10.7123
		2021-31	15.0143	15.4209	15.1952	14.1392	11.0985
		2031-41	15.1797	15.4861	15.2840	14.3034	11.3953
		2041-51	15.3247	15.5433	15.3638	14.4635	11.7110
		2051-61	15.4260	15.5836	15.4215	14.5889	11.9795
	Male	2011-21	14.6491	15.2127	14.8526	13.3785	10.0451
		2021-31	14.8703	15.3047	14.9798	13.5921	10.3471
		2031-41	15.0478	15.3788	15.0853	13.7832	10.6366
		2041-51	15.1900	15.4389	15.1730	13.9543	10.9133
		2051-61	15.3040	15.4879	15.2464	14.1076	11.1773
$\ddot{a}_{x:\overline{20}|}$	Female	2011-21	11.5994	12.1030	12.0278	11.7191	10.0201
		2021-31	11.7703	12.1439	12.0846	11.8199	10.2866
		2031-41	11.8773	12.1684	12.1189	11.8848	10.4823
		2041-51	11.9708	12.1891	12.1482	11.9437	10.6813
		2051-61	12.0359	12.2031	12.1681	11.9865	10.8429
	Male	2011-21	11.5520	12.0849	11.9740	11.4646	9.5127
		2021-31	11.6949	12.1194	12.0258	11.5673	9.7352
		2031-41	11.8089	12.1461	12.0665	11.6548	9.9415
		2041-51	11.8998	12.1667	12.0986	11.7294	10.1323
		2051-61	11.9721	12.1828	12.1241	11.7930	10.3083

Acknowledgement

Authors are very much indebted to anonymous referee for valuable suggestions and constructive comments on earlier version of paper which improve the quality and presentation of paper. First author would like to thanks to Department of Science and Technology (INSPIRE Division), New Delhi, India for financial assistance through INSPIRE fellowship for this research.

References

1. Aberathna, W., Alles, L., Wickremasinghe, W. N., and Hewapathirana, I. (2014). Modeling and Forecasting Mortality in Sri Lanka. *Sri Lankan Journal of Applied Statistics*, 15(3). DOI:10.4038/sljastats.v15i3.7794

2. Chiang, C. L. (1984). *The life Table and its applications*. Malabar: Robert E Krieger Publishing Company.

3. Greenwood, M. (1922). Discussion on the Value of Life-Tables in Statistical Research. *Journal of the Royal Statistical Society*, 85(4), 537. DOI:10.2307/2341122

4. Hyndman, R. J., and Khandakar, Y. (2008). Automatic Time Series Forecasting: The forecast Package for R. *Journal of Statistical Software*, 27(3). DOI:10.18637/jss.v027.i03

5. Koissi, M.-C., Shapiro, A. F., and Högnäs, G. (2006). Evaluating and extending the Lee–Carter model for mortality forecasting: Bootstrap confidence interval. *Insurance: Mathematics and Economics*, 38(1), 1–20. DOI:10.1016/j.insmatheco.2005.06.008

6. Lee, R. D., and Carter, L. R. (1992). Modeling and Forecasting U.S. Mortality. *Journal of the American Statistical Association*, 87(419), 659–671. DOI:10.1080/01621459.1992.10475265

7. Lee, R.D., and Rofman, R. (1994). Modeling and forecasting mortality in Chile. *Notas*, 22 (59), 182–213.

8. Li, N., Lee, R., and Tuljapurkar, S. (2004). Using the Lee-Carter Method to Forecast Mortality for Populations with Limited Data. *International Statistical Review*, 72(1), 19–36. DOI:10.1111/j.1751-5823.2004.tb00221.x

9. Li, S. H. and Chan, W. S. (2004). Estimation of complete period life Tables for Singaporeans. *Journal of Actuarial Practice*, 11, 129-146.

10. Lin, J. (1995). Changing Kinship Structure and its Implications for Old-Age Support in Urban and Rural China. *Population Studies*, 49(1), 127–145. DOI:10.1080/0032472031000148286

11. Singh, A. and Ram, F. (2004). *Forecasting mortality in India*. Rawat Publications, Jaipur, Population Health and Development. Ed. TK Roy, M Guruswamy and P Arokiasamy.

12. Tuljapurkar, S., Nan, L. and Boe, C., (2000). A universal pattern of mortality decline in the G7 countries. *Nature*, 405, 789–792.

13. Wilmoth, J.R., (1996). *Mortality Projections for Japan: A Comparison of Four Methods.* Health and Mortality among Elderly Population. In: Caselli, G., Lopez, A. (Eds.), Oxford University Press, New York.

14. Yadav, A., Yadav, S., and Kesarwani, R. (2012). Decelerating Mortality Rates in Older Ages and its Prospects through Lee-Carter Approach. *PLoS ONE,* 7(12): e50941. DOI:10.1371/journal.pone.0050941

15. Yasungnoen, N., and Sattayatham, P. (2016). Forecasting Thai Mortality by Using the Lee-Carter Model. *Asia-Pacific Journal of Risk and Insurance*, 10(1), 91-105. DOI:10.1515/apjri-2014-0042