Sri Lankan Journal of Applied Statistics, Vol (17-3)
DOI: http://doi.org/10.4038/sljastats.v17i3.7904

New Extended Burr Type X Distribution

Arwa Y. Al-Saiari[1] Lamya A. Baharith[1], Salwa A. Mousa[2*]

[1]Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia.

[2]Faculty of Commerce (Girls Campus), Al-Azhar University, Nasr City, Cairo, Egypt.

*Corresponding Author: salwa_elaty11@hotmail.com

Received: 19th November 2015/ Revised: 2nd March 2017/ Accepted: 20th March 2017

ABSTRACT

This study introduces a new extended distribution of the Burr Type X distribution, using Marshall-Olkin method called the Marshall-Olkin Extended Burr Type X distribution. This study formulates the new distribution by inducing a tilt parameter in the Burr Type X distribution of Marshall-Olkin, to account for the significance of the size of an event. Model parameters are obtained using maximum likelihood and Bayesian methods of estimation. The scale and shape parameters are specifically defined to identify the dimensions and density of an event. Mathematical and statistical properties and limitations of the distribution are also presented. Lifetime data analysis is performed to demonstrate the model's applicability and flexibility. Akaike and Bayesian Information Criteria illustrate that the new distribution provides better fit compared to other distributions,

Keywords: Extended Burr Type X, MOEBX Distribution, Tilt parameter, Marshall-Olkin, Maximum Likelihood Estimators, Bayesian Estimators, Skewed Distribution, Akaike, and Bayesian Information Criteria.

1. Introduction

Twelve different forms of distributions were introduced by Burr (1942), where the Burr Type X and Burr Type XII have gained significant interest from many researchers. Among them, Rodriguez (1977) and Wingo (1993) who focused on the examination of the Burr Type XII distribution. Also, Al-Saiari et al. (2014) proposed Marshall-Olkin Burr Type XII distribution. This research article particularly discusses the properties of Burr Type X distribution and its associated relationship with other distributions, to build the ground for lifetime data analysis. The Burr Type X distribution fundamentally includes two parameters with the cumulative distribution function (CDF) as follows

$$F(x;k,\lambda) = \left(1 - e^{-(\lambda x)^2}\right)^k ; \quad x > 0, k, \lambda > 0$$

where λ and κ are scale and shape parameters, respectively (Raqab & Kundu 2006).

Many researchers investigated the single- parameter model by, where λ=1. In this article, we will introduce the tilt parameter, which will result in a more flexible distribution compared to the original one.

For the single-parameter model, the probability density function (PDF) and CDF are written as:

$$f(x;k) = 2\,kxe^{-x^2}\left(1 - e^{-x^2}\right)^{k-1}; \quad x > 0, \quad k > 0$$

$$F(x;k) = \left(1 - e^{-x^2}\right)^k,$$

where, k is the shape parameter. Among the several studies that have been carried out on single-parameter Burr Type X distribution are; Ahmad Sartawi and Abu-Salih(1991), Jaheen (1995; 1996), Ahmad et al. (1997), Raqab (1998), and Surles and Padgett (1998). The single parameter distribution of the extended Burr Type X by Surles and Padgett (2001) considered as a generalized Rayleigh distribution. This is characterized by normally distributed random variables with a zero mean and a constant variance, and has no correlation, and therefore, making it more applicable, instead of restricting its applicability for lifetime data. For instance, this model can be applied to estimate the magnitude of noise variance in the Magnetic Resonance Imaging (Sijbers et al. 1999). Surles and Padgett(2005) derived the scaled Burr Type X distribution to fit the strength data, which was a comparative model to the Weibull distribution. Moreover, Raqab and Kundo (2006) developed a two-parameter Burr Type X distribution that has a closed form of the generalized Rayleigh distribution. This distribution was further compared with the Weibull exponentiated exponential, gamma, and generalized exponential distributions. Aludaat et al. (2008) used the Bayesian and non-Bayesian methods to estimate the parameters of Burr Type X distribution for Group Data.

New extended families of distribution were proposed with the inclusion of an additional parameter using Marshall and Olkin method which provide more flexible distribution and represented a wider range of behaviours (Marshall & Olkin 1997). Several researchers used this method to propose new extended distributions such as the extended Weibull compound distribution of Marshall Olkin by Ghitany et al. (2005), Pareto extended distribution of Marshall Olkin by Ghitany (2005), extended gamma distribution of Marshall Olkin by Ristic et al. (2007), extended Lomax distribution of Marshall Olkin for censored data by Ghitany et al. (2007), extended exponential distribution of Marshall Olkin by Srivastava et al. (2011), extended uniform distribution of Marshall Olkin by Jose and Krishna (2011), extended power lognormal distribution of Marshall Olkin by Gui(2013), negative binomial Marshall–Olkin Rayleigh distribution by Jose and Sivadas (2015), extended generalized linear exponential distribution of Marshall Olkin by Okasha and Kayid (2016), and extended Burr III distribution of Marshall Olkin by Al-Saiari et al. (2016).

This paper intends to study the properties of the Marshall Olkin extended Burr Type X (MOEBX) distribution. The paper first illustrates the construction of MOEBX in Section 2. Section 3 provides graphical illustrations of the dimensions of MOEBX distribution. Parameter estimation using Maximum Likelihood and Bayesian methods are presented, respectively, in Sections 4 and 5. Section 6 focuses on defining the statistical properties of MOEBX distribution along with application to real lifetime data. Finally, some concluding remarks are given in Section 7.

2. Construction of the Extended Burr Type X Distribution of Marshall-Olkin

To construct the survival function of the proposed MOEBX distribution, $\bar{G}(x;\,\alpha)$, we need to obtain the survival function, $\bar{F}(x) = 1 - F(x)$, of the Burr Type X distribution. This can be given by:

$$\bar{G}(x;\alpha) = \frac{\alpha\bar{F}(x)}{1-\bar{\alpha}\bar{F}(x)}, \quad -\infty < x < \infty, \quad 0 < \alpha < \infty, \qquad (1)$$

where, $F(x) = \left(1 - e^{-x^2}\right)^k$, $\bar{F}(x) = 1 - F(x)$, $\alpha = 1 - \bar{\alpha}$ and

α is the tilt parameter, which measures the shape of the rising boundaries (Jose 2011). The CDF and PDF for MOEBX distribution are respectively written as:

$$G(x;\alpha,k) = 1 - \frac{\alpha\bar{F}(x)}{1-\bar{\alpha}\bar{F}(x)} = \frac{\left(1-e^{-x^2}\right)^k}{\left[1-(1-\alpha)\left[1-\left(1-e^{-x^2}\right)^k\right]\right]} \quad (2)$$

and

$$g(x;\alpha,k) = \frac{\alpha f(x)}{[1-\bar{\alpha}\bar{F}(x)]^2} = \frac{2\,\alpha k x e^{-x^2}\left(1-e^{-x^2}\right)^{k-1}}{\left[1-(1-\alpha)\left[1-\left(1-e^{-x^2}\right)^k\right]\right]^2}; \quad (3)$$

where $x > 0, \alpha and k > 0$.

3. Properties of the Extended Burr Type X Distribution of Marshall-Olkin

Assume a random variable X that follows the MOEBX distribution, denoted by X ~ MOEBX (α, k). Figure 1 below depicts the cumulative and probability density functions of MOEBX distribution (α, k) at different values of its parameters, α and k.

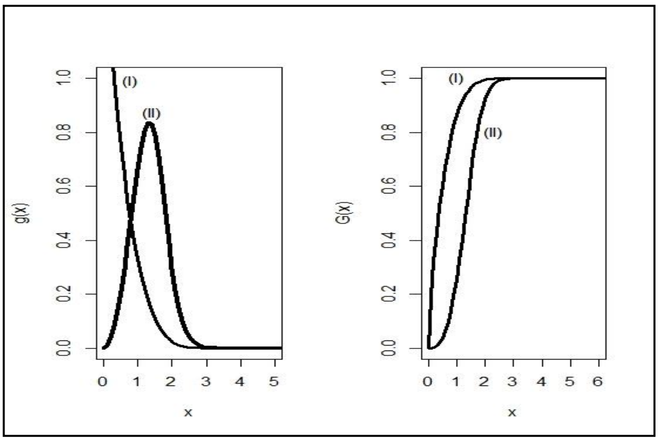

Figure 1: PDF and CDF of the MOEBX distribution: (I) α = 0.8, k = 0.4; (II) α = 0.3, k = 1.5

From Figure 1, it is clear that the PDF takes various shapes, depending on the values of α and k:

i. For k > 0.5 at varying values of α, the density function is unimodal, i.e. g(x; α, k) is skewed towards right
ii. For k < 0.5 and α < 1, the density function is decreasing, i.e. g(x; α, k) is L-shaped.

3.1 The Hazard Function

The hazard rate of the original Burr Type X distribution, h(x) is directly related to the hazard rate of the extended distribution of Marshall-Olkin r(x; α), which is given by:

$$r(x;\alpha) = \frac{h(x)}{1-\bar{\alpha}\bar{F}(x)} \tag{4}$$

In which, h(x) is the corresponding hazard rate of f(x).From the above equation (4),

$\frac{r(x,\alpha)}{h(x)}$ is a decreasing for all values of x when 0 < α < 1, and
$\frac{r(x,\alpha)}{h(x)}$ is increasing for all values of x when α > 1, (Jose 2011).
Therefore, the hazard rate function and the survival function of the MOEBX distribution are respectively given by

$$r(x;\alpha,k) = \frac{2\,kxe^{-x^2}\left(1-e^{-x^2}\right)^{k-1}}{\left\{1-(1-\alpha)\left[1-\left(1-e^{-x^2}\right)^{k}\right]\right\}\left[1-\left(1-e^{-x^2}\right)^{k}\right]} \tag{5}$$

$$\bar{G}(x;\alpha,k) = \frac{\alpha\left[1-\left(1-e^{-x^2}\right)^{k}\right]}{\left\{1-(1-\alpha)\left[1-\left(1-e^{-x^2}\right)^{k}\right]\right\}} \tag{6}$$

Marhall and Olkin (1997), and Cordeiro et al. (2014) studied the relationship between r(x; α, k) and $\bar{G}$(x; α, k) to determine the relationship between extended and original distribution, which depends on the range of the new parameter. From (5), it is clear that r(x; α, k) takes various shapes:

i. For $k<1$ at varying values of α, r(x; α, k) is monotone increasing.
ii. For $k>1$ at varying values of α, r(x; α, k) decreases after reaching its maximum, which can be clearly observed when α ≤ 1.

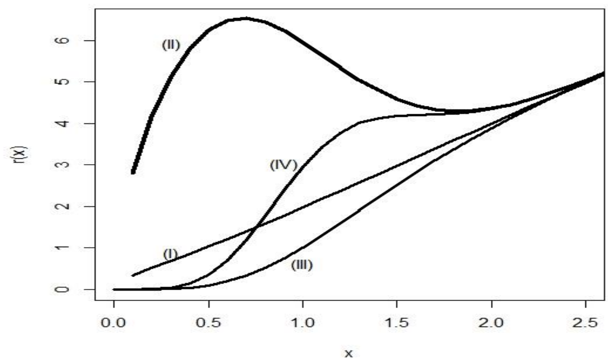

Figure 2: Hazard rate of the MOEBX distribution: (I) α = 0.8, k = 0.8; (II) α = 0.7, k = 0.8; (III) α = 0.8, k = 3; (IV) α = 0.3, k = 3

Moreover, the relationship depends on the range of the new parameters α > 1 or 0 < α < 1, (Marshall & Olkin 1997).
Thus, for 0 < α < 1 $and x > 0$, it can be shown that:

$$\frac{2\,kxe^{-x^2}\left(1-e^{-x^2}\right)^{k-1}}{\left[1-\left(1-e^{-x^2}\right)^{k}\right]} \le r(x;\alpha,k) \le \frac{2\,kxe^{-x^2}\left(1-e^{-x^2}\right)^{k-1}}{\alpha\left[1-\left(1-e^{-x^2}\right)^{k}\right]} \qquad (7)$$

$$\left[1-\left(1-e^{-x^2}\right)^{k}\right]^{1/\alpha} \le \bar{G}(x;\alpha,k) \le \left[1-\left(1-e^{-x^2}\right)^{k}\right] \qquad (8)$$

While for α > 1, it follows that:

$$\frac{2\,kxe^{-x^2}\left(1-e^{-x^2}\right)^{k-1}}{\alpha\left[1-\left(1-e^{-x^2}\right)^{k}\right]} \le r(x;\alpha,k) \le \frac{2\,kxe^{-x^2}\left(1-e^{-x^2}\right)^{k-1}}{\left[1-\left(1-e^{-x^2}\right)^{k}\right]} \qquad (9)$$

$$\left[1-\left(1-e^{-x^2}\right)^{k}\right] \le \bar{G}(x;\alpha,k) \le \left[1-\left(1-e^{-x^2}\right)^{k}\right]^{1/\alpha} \qquad (10)$$

3.2 The Quantile, Median, and Mode

The qth quantile of the MOEBX distribution is:

$$x_q = G^{-1}(q) = \left[\log\left[1-\left[1+\left(\frac{1-q}{\alpha q}\right)\right]^{-1/k}\right]^{-1}\right]^{1/2};\ \ 0 \le q \le 1, \quad (11)$$

where $G^{-1}(q)$ represents the inverse function of the MOEBX distribution. Then replacing q with 0.5, the median of the MOEBX distribution is:

$$median = \left[\log\left[1 - \left(\frac{\alpha}{1+\alpha}\right)^{1/k}\right]^{-1}\right]^{1/2}. \quad (12)$$

Solving the equation $\frac{d\log g(x;\alpha,k)}{dx}$, the mode of MOEBX distribution can be found by:

$$\frac{d\log g(x;\alpha,k)}{dx} = \frac{1}{x} - 2x + 2(k-1)\frac{xe^{-x^2}}{\left(1-e^{-x^2}\right)}$$

$$-4(1-\alpha)k\frac{(1-e^{-x^2})^{k-1}xe^{-x^2}}{\left[1-(1-\alpha)\left[1-\left(1-e^{-x^2}\right)^k\right]\right]} = 0 \quad (13)$$

Table 1 reports the modes for some given values of k and α, which illustrates that the PDF of MOEBX distribution is unimodal for these values of the parameters.

Table 1: The modes for MOEBX distribution for given values of parameters

Parameters	Mode
k=0.4, α =1.2	0.3109
k=1.5, α =1. 2	0.9789

4. Maximum Likelihood Estimation

Maximum likelihood (ML) method is used to obtain the parameters of the MOEBX distribution. Assuming that we have a random sample $\underline{X} = (X_1, X_2, \ldots . X_n)$ of size n from MOEBX distribution, the log-likelihood of MOEBX is given by:

$$l(\alpha,k) = nlog2 + nlog\alpha + nlogk + \sum_{i=1}^{n}\log(x_i) - \sum_{i=1}^{n} x_i^2 + (k-1)\sum_{i=1}^{n} log\big(\varphi_3(x_i)\big) - 2\sum_{i=1}^{n} log\big(\omega_3(x_i;\alpha,k)\big),\ \text{i} = 1, \dots, \text{n} \qquad (14)$$

where

$$\varphi_3(x_i) = \left(1 - e^{-x_i{}^2}\right), \omega_3(x_i;\alpha,k) = 1 - (1-\alpha)\left[1 - (1 - e^{-x_i{}^2})^k\right] \quad (15)$$

ML estimates of θ denoted by $\hat{\theta}_{ML} = (\hat{\alpha}_{ML}, \hat{k}_{ML})$ can be obtained by maximizing (14) using iterative procedure to solve the following non-linear equations

$$\frac{\partial l(\hat{\alpha},\hat{k})}{\partial\hat{\alpha}} = \frac{n}{\hat{\alpha}} - 2\sum_{i=1}^{n}\frac{[1-[\varphi_3(x_i)]^{\hat{k}}]}{\omega_3(x_i;\hat{\alpha},\hat{k})} = 0 \qquad (16)$$

$$\frac{\partial l(\hat{\alpha},\hat{k})}{\partial k} = \frac{n}{\hat{k}} + \sum_{i=1}^{n} log\varphi_3(x_i) - 2(1-\hat{\alpha})\sum_{i=1}^{n}\frac{[\varphi_3(x_i)]^{\hat{k}} log\varphi_3(x_i)}{\omega_3(x_i;\hat{\alpha},\hat{k})} = 0 \qquad (17)$$

Approximate Confidence Intervals:

It is clear that we cannot obtain the ML estimates of the parameters of MOBEX distribution in close form. For this reason, approximations are used to compute the confidence interval of $\bar{\theta}$, the estimates of which are rooted in the asymptotic distribution of ML approach. This is based on the following assumptions:

- large size of the sample is taken for the estimations of $\bar{\hat{\theta}}_{ML}$,
- Estimates follow a normal distribution i.e.
- mean $\bar{\theta} = (\alpha, k)$,
- a constant variance, and
- Variance-covariance matrix of I^{-1} (Inverse matrix of Fisher Information).

Then using this approach, the confidence interval of the parameters at the level of $100\left(1-\frac{\gamma}{2}\right)$% will be approximated as:

$$\hat{\alpha}_{ML} \pm z_{\gamma/2}\sqrt{var(\hat{\alpha}_{ML})} and \hat{k}_{ML} \pm z_{\gamma/2}\sqrt{var\big(\hat{k}_{ML}\big)}, \qquad (18)$$

where $z_{\gamma/2}$ is the standard normal distribution with $\gamma/2^{\text{th}}$ percentile.

In order to evaluate whether the MOEBX distribution fits better on the data compared to the Burr Type X distribution, the Likelihood ratio (LR) statistical test is conducted. That is, the null hypothesis $H_0: \alpha = 1$ is tested against $H_1: \alpha \neq 1$ and the test statistic of is computed by:

$$\Lambda = -2\ln\lambda = -2\ln\left[\frac{L_0(\tilde{\theta})}{L_1(\hat{\theta})}\right] = -2\left[\ln L_0(\tilde{\theta}) - \ln L_1(\hat{\theta})\right],$$

where $L_0(\tilde{\theta})$ is the ML estimate of $\bar{\theta}$ under H_0, $\Lambda \sim \chi^2_{1-\alpha,1}$.

5. Bayesian Estimation

Suppose that for a random sample $\underline{X} = (X_1, X_2, \ldots . X_n)$ of size n from MOEBX distributionwith the unknown parameters $\bar{\theta} = (\alpha, k)$ which are assumed to be independent having non-informative prior distributions such that α ~uniform (a_1,b_1) and k ~uniform (a_2,b_2). Then the joint prior for α and k can be written by

$$\pi(\alpha, k) = \pi_1(\alpha)\pi_2(k) = \frac{1}{\alpha} \times \frac{1}{k}, \quad \alpha, k > 0$$

Therefore, the subsequent joint posterior distribution can be written as

$$\pi(\alpha, k|\underline{x}) = A\alpha^n \mathrm{k}^n \frac{2^n \prod_{i=1}^n (x_i) e^{-\sum_{i=1}^n {x_i}^2} \prod_{i=1}^n [\varphi_3(x_i)]^{k-1}}{\prod_{i=1}^n [\omega_3(x_i;\alpha,k)]^2} \pi(\alpha, k), \qquad (19)$$

where $\varphi_3(x_i), \omega_3(x_i; \alpha, k)$are as defined in (15), and A is the normalizing constant, which ensures that the area represents the probability density function. Markov Chain Monte Carlo technique can be used to obtain the posterior estimates of the parameters for MOEBX under squared error loss function.

6. Application to Real Life Data

Lawless (2011) described the breaking strength T of single carbon fibers of different lengths which consisted of 70 observations. The Kolmogorov-Smirnov (K-S) statistic and the empirical CDF were used to validate the fitted model. The empirical CDF and the fitted CDF is shown in Figure 3 by replacing the parameters with their ML estimates. The K-S test statistic is

0.1571 with the p-value = 0.3531 using ML estimates. When using Bayesian estimates the K-S test value is 0.2 with the p-value = 0.1216. Therefore, both K-S tests values and Figure 3 indicate that MOEBX distribution provides the appropriate fit for this data set.

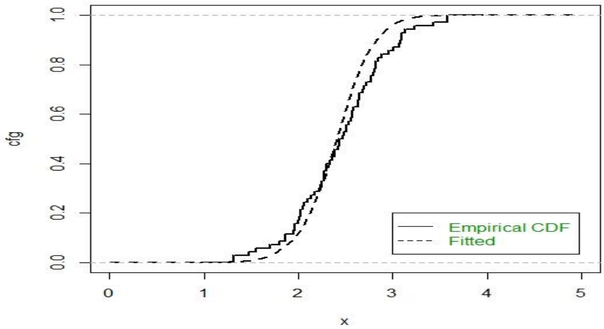

Figure 3: The plot for the fitted and empirical CDF of the MOEBX distribution

The ML and Bayesian estimates of the MOEBX parameters, standard error (SE), 95% confidence intervals (CI), and interval length are illustrated in Table 2.

Table 2: ML and Bayesian estimates of MOEBX parameters, SE, CI and interval length.

Parameter	ML			Bayesian		
	Estimate	SE	95% CI	Estimate	SE	95% CI
k	19.73	0.028	(19.68, 20.28) (0.61)	19.54	0.005	(19.54,19.5) (0.01)
α	16.50	0.205	(16.10,16.90) (0.80)	14.84	0.005	(14.83,14.8) (0.02)

To examine the flexibility of the proposed MOEBX distribution, the same set of real data has been used to fit the model, using ML and Bayesian methods of estimation. ML and Bayesian estimates, Akaike information criterion (AIC) and Bayesian information criterion (BIC) are reported in Table 3 in order to compare the two distributions. This was conducted using Algorithms 1(see Appendix).

The LR test statistics for testing the hypotheses $H_0: \alpha = 1$ versus $H_1: \alpha \neq 1$ is Λ=146.33 (p-value < 0.05) which indicates that the MOEBX distribution is more suitable than Burr Type X distribution for this particular data set.

Table 3: ML and Bayesian estimates of MOEBX parameters along with AIC and BIC statistics.

Distribution	ML		Statistic		Bayesian		Statistic	
	$\hat{k}$	$\hat{\alpha}$	AIC	BIC	$\tilde{k}$	$\tilde{\alpha}$	AIC	BIC
Burr X	22.50	-	377.88	380.13	20.11	-	377.88	380.13
MOEBX	19.73	16.50	121.28	125.78	19.54	14.84	122.49	126.99

The results indicate that the MOEBX distribution, in comparison with the Burr X distribution, has by far the lowest statistics of AIC and BIC values. Thus, it can be asserted that MOEBX distribution is more flexible than the Burr Type X distribution. Moreover, the normal quantile-quantile plot shows that the error of the fitted MOEBX distribution tends to approximate normality, (See Figure 4).

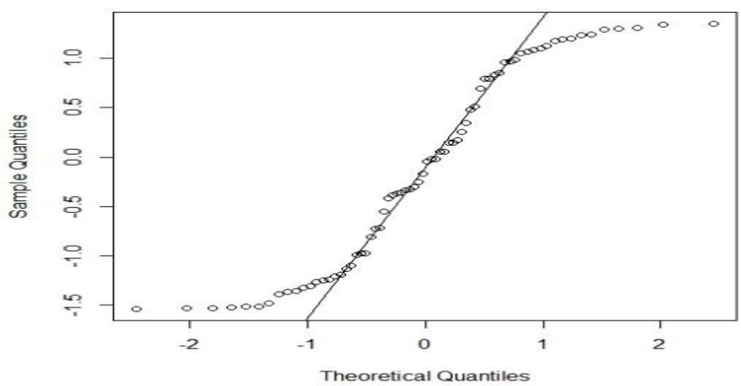

Figure 4: Normal Quantile-Quantile Plot of Fitted MOEBX Distribution.

7. Conclusion

This paper aimed to propose a MOEBX distribution using Marshall-Olkin method by adding a tilt parameter as an extension to the Burr Type X distribution. The mathematical and statistical properties of the MOEBX distribution are discussed and its parameters are estimated using ML and Bayesian methods. The confidence interval for the parameters is asymptotic and based on certain assumptions regarding the mean, variance and variance-covariance matrix. A realife data set was fitted to the proposed distribution to illustrate the flexibility of the MOEBX distribution. Parameter estimates were obtained using ML and Bayesian methods. The standard errors of the estimates and lengths of the intervals showed that the Bayesian estimates provide a better fit for the data than the ML estimates.

Acknowledgements

The authors are very grateful to the reviewers and the editor for their valuable comments and constructive suggestions which certainly improved the quality of the paper in the present version.

References

1. Ahmad, K.E., Fakhry, M.E. & Jaheen, Z.F., (1997). Empirical Bayes estimation of P (Y< X) and characterizations of Burr-type X model. Journal of Statistical Planning and Inference, 64(2), pp.297–308.DOI: 10.1016/S0378-3758(97)00038-4
2. Ahmad Sartawi, H. & Abu-Salih, M.S., (1991). Bayesian prediction bounds for the Burr type X model. Communications in Statistics-Theory and Methods, 20(7), pp.2307–2330.DOI: 10.1080/03610929108830633
3. Al-Saiari, A.Y., Baharith, L.A. & Mousa, S.A., (2014). Marshall-Olkin Extended Burr Type XII Distribution. International Journal of Statisticsand probability, 3(1), p.78-84.DOI:org/10.5539/ijsp.v3n1p78
4. Al-Saiari, A.Y., Mousa, S.A. & Baharith, L.A., (2016). Marshall-Olkin Extended Burr III Distribution. International Mathematical Forum, 11(13), pp.631–642. Available at: http://www.m-hikari.com/imf/imf-2016/13-16-2016/p/baharithIMF13-16-2016.pdf.DOI: 10.12988 /imf.2016.6554

5. Aludaat, K.M., Alodat, M.T. & Alodat, T.T., (2008). Parameter estimation of Burr type X distribution for grouped data. Applied Mathematical Sciences, 2(9), pp.415–423.DOI: 10.1214/aoms/1177731607.
6. Burr, I.W., (1942). Cumulative frequency functions. The Annals of mathematical statistics, 13(2), pp.215–232.DOI:org/10.1214/aoms/1177731607
7. Cordeiro, G.M., Lemonte, A.J. & Ortega, E.M.M., (2014). The Marshall--Olkin Family of Distributions: Mathematical Properties and New Models. Journal of Statistical Theory and Practice, 8(2), pp.343–366.DOI:org/10.1080/15598608.2013.802659
8. Ghitany, M.E., (2005). Marshall-Olkin extended Pareto distribution and its application. International Journal of Applied Mathematics, 18(1), p.17.DOI:10.1080/02664760500165008
9. Ghitany, M.E., Al-Awadhi, F.A. & Alkhalfan, L.A., (2007). Marshall--Olkin extended Lomax distribution and its application to censored data. Communications in Statistics—Theory and Methods, 36(10), pp.1855–1866.DOI:org/10.1080/03610920601126571
10. Ghitany, M.E., Al-Hussaini, E.K. & Al-Jarallah, R.A., (2005). Marshall--Olkin extended Weibull distribution and its application to censored data. Journal of Applied Statistics, 32(10), pp.1025–1034.DOI:.org/10.1080/02664760500165008
11. Gui, W., (2013). A Marshall-Olkin power log-normal distribution and its applications to survival data. International Journal of Statistics and Probability, 2(1), p.63-72.DOI:org/10.5539/ijsp.v2n1p63
12. Jaheen, Z.F., (1995). Bayesian approach to prediction with outliers from the Burr type X model. Microelectronics Reliability, 35(4), pp.703–705.DOI:org/10.1016/0026-2714(94)00076-Z
13. Jaheen, Z.F., (1996). Empirical Bayes estimation of the reliability and failure rate functions of the Burr type X failure model. Journal of Applied Statistical Science, 3(4), pp.281–288.DOI: 10.1016/0026-2714(94)00056-T.
14. Jose, K.K., (2011). Marshall-Olkin Family of Distributions and

their applications in reliability theory, time series modeling and stress-strength analysis. Proc. ISI 58th World Statist. Congr Int Stat Inst, 21st-26th August, pp.3918–3923.
15. Jose, K.K. & Krishna, E., (2011). Marshall-Olkin extended uniform distribution. In ProbStat Forum. pp. 78–88.
16. Jose, K.K. & Sivadas, R., (2015). Negative binomial Marshall--Olkin Rayleigh distribution and its applications. Economic Quality Control, 30(2), pp.89–98.DOI: 10.1515/eqc-2015-0009.
17. Lawless, J.F., 2011. Statistical models and methods for lifetime data, John Wiley & Sons.
18. Marshall, A.W. & Olkin, I., (1997). A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families. Biometrika, 84(3), pp.641–652.DOI: 10.1093/biomet/84.3.641.
19. Okasha, H.M. & Kayid, M., (2016). A new family of Marshall--Olkin extended generalized linear exponential distribution. Journal of Computational and Applied Mathematics, 296, pp.576–592.DOI:10.1016/j.cam.2015.10.017
20. Raqab, M.Z., (1998). Order statistics from the Burr type X model. Computers & Mathematics with Applications, 36(4), pp.111–120.
21. Raqab, M.Z. & Kundu, D., (2006). Burr type X distribution: revisited. Journal of probability and statistical sciences, 4(2), pp.179–193.
22. Ristic, M.M., Jose, K.K. & Ancy, J., (2007). A Marshall--Olkin gamma distribution and minification process. Stress Anxiety Res Soc, 11, pp.107–117.
23. Rodriguez, R.N., (1977). A guide to the Burr type XII distributions. Biometrika, pp.129–134.DOI:org/10.1093/biomet/64.1.129
24. Sijbers,J., Dekker,A.J.,Raman,E. & Van Dyck,D.,(1999). Parameter estimation from magnitude MR images. International Journal of Imaging Systems and Technology, 10(2), pp.109–114.DOI:org/10.1002/(SICI)1098-1098(1999)10:2<109::AID-IMA2>3.0.CO;2-R
25. Srivastava, A.K. & Kumar, V., (2011). Software reliability data analysis with Marshall-Olkin Extended Weibull model using

MCMC method for non-informative set of priors. International Journal of Computer Applications, 18(4), pp.31–39.DOI:org/10.5120/2271-2926
26. Surles, J.G. & Padgett, W.J., (1998). Inference for P (Y< X) in the Burr type X model. Journal of Applied Statistical Science, 7(4), pp.225–238.
27. Surles, J.G. & Padgett, W.J., (2001). Inference for reliability and stress-strength for a scaled Burr Type X distribution. Lifetime Data Analysis, 7(2), pp.187–200.DOI:.org/10.1023/A:1011352923990
28. Surles, J.G. & Padgett, W.J., (2005). Some properties of a scaled Burr type X distribution. Journal of Statistical Planning and Inference, 128(1), pp.271–280.DOI:org/10.1016/j.jspi.2003.10.003
29. Wingo, D.R., (1993). Maximum likelihood methods for fitting the Burr type XII distribution to multiply (progressively) censored life test data. Metrika, 40(1), pp.203–210.DOI: 10.1007/ BF02613681.

Appendix

Algorithm 1

1. Plot the CDF of the MOEBX Vs the empirical distribution of the data to obtain initial values of the parameters.
2. Obtain ML and Bayesian estimates of the MOEBX parameters based on the data, using nlminb and Metro_Hastings functions in R program, respectively.
3. Calculate Kolmogorov–Smirnov (K–S) goodness of fit test based on ML and Bayesian estimates obtained in step 2 to validate the appropriateness of the fitted model to the data.
4. Obtain ML and Bayesian estimates of the original distribution, BURR X, parameters based on the data, using nlminb and Metro_Hastings functions in R program, respectively.
5. Compute AIC and BIC statistics for both MOEBX and BURR X models based on ML and Bayesian estimates to compare between the two models.