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Improved Estimation of Sensitive Mean Using Hybrid of Partial and Optional Scrambling in the Presence of Non-Sensitive Auxiliary Information

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ABSTRACT

This article is about studying ratio, product and regression methods for estimating sensitive mean using a two-stage optional randomized response model by Gupta et al. (2010) and information on non-sensitive auxiliary variable. In particular, the additive randomized response model is used to further enhance the efficiency of the ratio, product and regression estimators (Gupta et al., 2010). We compare our proposed auxiliary information based two-stage optional randomized response estimator with recently proposed auxiliary information-based estimators. Through algebraic comparisons, it is shown that the proposed ratio, product and regression estimators are better than the corresponding estimators proposed in some recent studies. The results are also supported by a numerical study.

Key words: Two-stage optional randomized response model, mean squared error, ratio estimator, product estimator and regression estimator.

1. Introduction

Since long, the sample survey has been appreciated as a go to method for the procurement of data from several areas of human interest. For instance, in political science, Cronin Jr. et al. (2000) recognized the convenience, handiness, functionality and practicality of human surveys and mentioned it as a mostly used method for understanding politics. To have more insight about this research area

One may refer to compelling accounts of Mahoney and Rueschemeyer (2003) and McNabb (2015). Other than this, a rich number of applications of surveys can be observed in different fields of interest like demography (Kellogg (2014) and Abel, et al. (2016)), policy making (Eurostate – household income and expenditure survey and U.S. Bureau of Justice Statistics surveys), epidemiology (Dawidowska et al. (2012) and Ojikutu and Bolanle (2018)), psychology (Karelaia and Hogarth (2008) and Robinson and Irwin (2019)) etc.

While studying stigmatizing behaviors, avoiding the social desirability bias (SDB) is the main challenge for survey practitioners. The main reason for observing the SDB is the natural tendency of respondents to show possession of socially approved (accepted) traits. For a more detailed explanation of this issue, one may refer to Groves et al. (2009), Kelly et al. (2013) and Schill and Kirk (2017). Some of the survey practitioners have also noted troublesome points about the negative effect of SDB on the correctness of results obtained through surveys. Some of them are noted here as Heijden et al. (2000) observed that 75% of the respondents who had committed welfare or unemployment benefit fraud denied having done so in face-to-face interviews; Lee and Sargeant (2011) noted that 65% of the respondents over-reported their donations; Stecklov et al. (2015), observed that high reported use of sterilization is correlated with propensity of respondents to present themselves in a positive way in front of interviewers.

Amongst many, the most effective approach to deal with the problem of SDB is the randomized response techniques (RRT), initially, presented by Warner (1965). Over the time, it was further developed and applied by Greenberg et al. (1969), Horvitz et al. (1967), Himmelfarb and Edgell (1980), Chaudhuri and Mukerjee (1985), Kuk (1990), Mangat and Singh (1990), Mangat (1994), Tracy and Mangat (1996), Mahmood et al. (1998), Bhargava and Singh (2000), Gupta et al. (2002), Singh et al. (2003), Bar-Lev et al. (2004), Ryu et al. (2006), Gupta et al. (2006), Gjestvang and Singh (2007), Huang (2008, 2010), and many others. Recently, Kuokkanen et al. (2017) praised RRTs as *"more sophisticated methods for SDB reduction"*. Further, Lensvelt-Mulders et al. (2005) in their extensive validity examination of 38 RRTs noted that *"the more sensitive the topic under investigation, the higher the validity of RR results"*. For further details, comprehending the effectiveness of RRTs, in study of sensitive attributes, one may also refer the works done by Blume et al. (2007), Krumpal (2012) , Cruyff et al. (2016) and Shah, et al. (2020a,b).

Over the years, it has been established that the use of non-sensitive auxiliary information is very helpful in increasing the efficiency of the estimates. In the conventional randomization method, scrambled information on only the sensitive study variable is collected. More recently, auxiliary information in sensitive surveys has been used quiet fruitfully by many authors like ZaiZai (2005–2006) in ratio type estimation of population proportion based on Warner (1965) model; Kadilar and Cingi (2006), Diana and Perri (2007), Turgut and Cingi (2008), Koyuncu and Kadilar (2009) and Shabbir and Gupta (2010) in ratio, product and regression estimation of mean of a sensitive variable. Motivated by the successful applications of auxiliary information in sensitive surveys, we intend to make use of auxiliary information along with a two-stage optional randomized response model of Gupta et al. (2010) to suggest ratio, product and regression estimators. Specifically, the intended ratio, product and regression estimators are based on one non-sensitive auxiliary variable. In order to have a comparative study, we plan to derive the expressions for bias and mean squared error (MSE) of proposed ratio, product and regression estimators using two-stage optional randomized responses and compare them with different corresponding ratio, product and regression estimators.

In rest of the article, we briefly present, in section 2, some background information on ratio, product and regression estimators based on non-sensitive auxiliary information and scrambled information, obtained through different RRTs, on sensitive variable. In section 3, we discuss a two-stage optional randomized response model to be used later. In section 4, firstly, we propose ratio, product and regression estimators and then find the expressions for bias and MSE of the proposed estimators. In section 5, we compare, numerically and theoretically, the proposed two-stage optional randomized response-based estimators with already existing recent estimators and establish the unconditional superiority of the proposed estimators. Concluding remarks are given in the section 6.

2. Some recent RRTs and associated estimators

In this section, we discuss some non-sensitive auxiliary information based on randomized response estimators suggested by Sousa et al. (2010), Gupta et al. 2012) and Gupta et al. (2014).

2.1. Additive scrambling model (Sousa et al. , 2010)

Let X be a sensitive study variable, Y be a non-sensitive auxiliary variable and S be a scrambling variable. As study variable is sensitive in nature, it cannot be observed directly. Sousa et al. (2010) used an additive scrambled response model where each respondent, selected in a random sample of size *n* drawn from a finite population, is asked to report a scrambled response, say Z given by $Z = X + S$ and true response on the auxiliary variable Y.

Let $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$ and $\bar{z} = \frac{\sum_{i=1}^{n} z_i}{n}$ be the sample means.

Also, let $E(X) = \mu_X$, $E(Y) = \mu_Y$, $E(Z) = \mu_Z$ and $E(S) = \mu_S = 0$ be the population means.

The mean μ_Y is assumed to be known and $E(X) = E(Z)$. The usual mean per unit unbiased estimator of μ_X, suggested by Sousa et al. (2010), is given by

$$\hat{\mu}_{XS} = \bar{z}.$$

Its variance is given by

$$Var(\hat{\mu}_{XS}) = MSE(\hat{\mu}_{XS}) = \left(\frac{1-f}{n}\right)\left(\sigma_X^2 + \sigma_S^2\right),$$

where $f = \frac{n}{N}$, $\sigma_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_X)^2$ and $\sigma_S^2 = \frac{1}{N-1}\sum_{i=1}^{N}(s_i - \mu_S)^2$. Sousa et al. (2010) also suggested a ratio estimator of μ_X which is given by

$$\hat{\mu}_{RS} = \left(\frac{\bar{z}}{\bar{y}}\right)\mu_Y.$$

The bias and MSE of the $\hat{\mu}_{RS}$ are given by

$$Bias(\hat{\mu}_{RS}) \cong \left(\frac{1-f}{n}\right)\mu_X (C_Y^2 - \rho_{YZ} C_Z C_Y)$$

$$MSE(\hat{\mu}_{RP}) \cong \left(\frac{1-f}{n}\right)\mu_X^2 (C_Z^2 + C_Y^2 - 2\rho_{YZ} C_Z C_Y), \qquad (1)$$

Where $\rho_{YZ} = \frac{\rho_{XY}}{\sqrt{1 + \frac{\sigma_S^2}{\sigma_X^2}}}$ the correlation coefficient between Y and Z.

$C_Z^2 = C_X^2 + \frac{\sigma_S^2}{\mu_X^2}$ is the squared coefficient of variation of *Z***.**

Using the same additive scrambled response model of Sousa et al. (2010), Gupta et al. (2012) suggested regression type estimator of the mean μ_X , given by

$$\hat{\mu}_{\text{Re}gG} = \bar{z} + \hat{\beta}_{ZY}(\mu_Y - \bar{y}),$$

where $\hat{\beta}_{ZY}$ is the sample regression coefficient between Z and Y .The expressions for the bias and MSE of the regression estimator are given as

$$Bias(\hat{\mu}_{\text{Re}gG}) \cong -\beta_{ZY}\left(\frac{1-f}{n}\right)\left\{\frac{\mu_{12}}{\mu_{11}} - \frac{\mu_{03}}{\mu_{02}}\right\},$$

Where $\beta_{ZY} = \beta_{XY}, f = \frac{n}{N}$ and $\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}(z_i - \bar{Z})^r(y_i - \bar{Y})^s.$

$$MSE(\hat{\mu}_{\text{Re}gG}) \cong \left(\frac{1-f}{n}\right)\mu_Z^2 C_Z^2\left\{1-\rho_{YZ}^2\right\}, \tag{2}$$

2.2. Gupta et al. (2014) optional additive scrambling model

Gupta et al. (2014) used an optional randomized response to estimate the mean of a sensitive variable. The i^{th} respondent in a sample of size *n* is requested to provide an additive scrambled response $Z' = X + SV$, where V is a Bernoulli random variable taking value 1(0), if the study variable is sensitive (insensitive) to a particular respondent. The respondents are also requested to give true response on the auxiliary variable. The usual unbiased estimator of the population mean is given by

$$\hat{\mu}_{XW} = \bar{z}'.$$

The MSE of the estimator $\hat{\mu}_{XW}$ is given by

$$Var(\hat{\mu}_{XW}) = MSE(\hat{\mu}_{XW}) = \left(\frac{1-f}{n}\right)\left(\sigma_X^2 + W\sigma_S^2\right),$$

where f, σ_X^2 and σ_S^2 are defined as above.

The ratio estimator, suggested by Gupta et al. (2014), is given by

$$\hat{\mu}_{RWG} = \left(\frac{\bar{z}'}{\bar{y}}\right)\mu_Y.$$

The bias and MSE of $\hat{\mu}_{RWG}$ are given by

$$Bias(\hat{\mu}_{RWG}) \cong \left(\frac{1-f}{n}\right)\mu_{Z'}(C_Y^2 - \rho_{YZ'}C_{Z'}C_Y)$$

$$MSE(\hat{\mu}_{RWG}) \cong \left(\frac{1-f}{n}\right)\mu_{Z'}^2\left(C_{Z'}^2 + C_Y^2 - 2\rho_{YZ'}C_{Z'}C_Y\right), \quad (3)$$

Where $\rho_{YZ'} = \dfrac{\rho_{XY}}{\sqrt{1+W\dfrac{\sigma_S^2}{\sigma_X^2}}}$, and $C_{Z'}^2 = C_X^2 + W\dfrac{\sigma_S^2}{\mu_X^2}$.

The regression estimator, suggested by Gupta et al. (2014), is given by

$$\hat{\mu}_{\mathrm{Re}gWG} = \bar{z}' + \hat{\beta}_{Z'Y}\left(\mu_Y - \bar{y}\right),$$

where $\hat{\beta}_{Z'Y}$ is the sample regression coefficient between Z' and Y.

The bias and MSE of the regression estimator $\hat{\mu}_{\mathrm{Re}gWG}$ are given by

$$Bias(\hat{\mu}_{\mathrm{Re}gWG}) \cong -\beta_{Z'Y}\left(\frac{1-f}{n}\right)\left\{\frac{\mu_{12}}{\mu_{11}} - \frac{\mu_{03}}{\mu_{02}}\right\},$$

$$MSE(\hat{\mu}_{\mathrm{Re}gWG}) \cong \left(\frac{1-f}{n}\right)\mu_{Z'}^2 C_{Z'}^2\left\{1-\rho_{YZ'}^2\right\}, \quad (4)$$

where $\rho_{YZ'}$ and $C_{Z'}^2$ are defined as above. Gupta et al. (2014) compared their regression estimator of mean with the regression estimator suggested by Gupta et al. (2012) and showed that their estimator performed better when $\rho_{XY}^2 > 0$.

Under the same optional RRT, if we apply the product estimation of mean, then the product estimator may be suggested as

$$\hat{\mu}_{PWG} = \left(\frac{\bar{y}}{\mu_Y}\right)\bar{z}',$$

with bias and MSE given by

$$Bias(\hat{\mu}_{PWG}) \cong \left(\frac{1-f}{n}\right)\mu_{Z'}\rho_{YZ'}C_{Z'}C_Y,$$

$$MSE(\hat{\mu}_{PWG}) \cong \left(\frac{1-f}{n}\right)\mu_{Z'}^2\left(C_{Z'}^2 + C_Y^2 + 2\rho_{YZ'}C_{Z'}C_Y\right). \quad (5)$$

3. Two Stage Optional Randomized Response Model

Gupta et al. (2010) proposed a two-stage optional RRT. In this technique, a random sample of size *n* is drawn from population of size *N*. As discussed earlier, the study variable X and the auxiliary variable Y are correlated and the correlation between X and Y is denoted by ρ_{XY} . Also, the scrambling variable S is assumed to have mean $\mu_S = 0$ and known variance σ_S^2 . The whole procedure was performed in two stages. At 1st stage, a randomly selected fixed proportion (T) of the selected respondent give the true response X and the remaining proportion $(1-T)$ of the Respondents is directed to go to stage 2. At stage 2, respondents were given an option to additively scramble their response if they feel study variable sensitive, otherwise, they were asked to report the true response on the sensitive variable. The distribution of the reported response, say Z^*, is given by:

$$Z^* = \begin{cases} X & \text{with probability } T + (1-T)(1-W) \\ S + X & \text{with probability } (1-T)W, \end{cases}$$

where W is named as the sensitivity level. The expected response may be written as

$$E(Z^*) = \mu_X + (1-T)W\mu_S.$$

Now, after replacing the value of $\mu_S = 0$, we get

$$E(Z^*) = \mu_X .$$

The usual unbiased estimator of the μ_X , suggested by Gupta et al. (2010), is given by $\hat{\mu}_{XP} = \bar{z}^*$,

Where $\bar{z}^* = \frac{\sum_{i=1}^{n} z_i^*}{n}$ and

its variance is given by

$$Var(\hat{\mu}_{XP}) = MSE(\hat{\mu}_{XP}) = \left(\frac{1-f}{n}\right)\left(\sigma_X^2 + W(1-T)\sigma_S^2\right), \tag{6}$$

4. Proposed Ratio, Product and Regression estimators

We now suggest the ratio, product and regression estimators under the Gupta et al. (2010) RRT. The reason behind taking Gupta et al. (2010) RRT is the advantage of distributing the total probability of reporting on sensitive variable into two stages which, in turn, results in more cooperation from the respondents in terms of truthful reporting.

4.1 Ratio Estimator

Now, we propose the ratio estimator of the population mean which is based on the true information on one non-sensitive auxiliary variable Y. The proposed ratio estimator of mean is given by

$$\hat{\mu}_{RP} = \left(\frac{\bar{z}^*}{\bar{y}}\right)\mu_Y.$$

The estimator $\hat{\mu}_{RP}$ can also be written as

$$\hat{\mu}_{RP} = \mu_{z^*}(1+e_{Z^*})(1+e_Y)^{-1},$$

Where $e_{Z^*} = \frac{\bar{z}^* - \mu_{Z^*}}{\mu_{Z^*}}$ and $e_Z = \frac{\bar{y} - \mu_Y}{\mu_Y}$.

Using Taylor's series expansion and first order approximation, we have

$$\hat{\mu}_{RP} - \mu_Z \cong \mu_{Z^*}\left(e_{Z^*} - e_Y - e_Y e_{Z^*} + e_Y^2\right),$$

The Bias and MSE of $\hat{\mu}_{RP}$ are given by

$$Bias(\hat{\mu}_{RP}) \cong \left(\frac{1-f}{n}\right)\mu_{Z^*}\left(C_Y^2 - \rho_{YZ^*} C_{Z^*} C_Y\right)$$

$$MSE(\hat{\mu}_{RP}) \cong \left(\frac{1-f}{n}\right)\mu_{Z^*}^2 (C_{Z^*}^2 + C_Y^2 - 2\rho_{YZ^*} C_{Z^*} C_Y),$$

Where $E(e_{Z^*}) = E(e_Y) = 0$, $E(e_{Z^*}^2) = \left(\frac{1-f}{n}\right) C_{Z^*}^2$, $E(e_Y^2) = \left(\frac{1-f}{n}\right) C_Y^2$

$E(e_{Z^*} e_Y) = \left(\frac{1-f}{n}\right)\rho_{YZ^*} C_{Z^*} C_Y$, $\rho_{YZ^*} = \dfrac{\rho_{XY}}{\sqrt{1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}}}$ and

$C_{Z^*}^2 = C_X^2 + W(1-T)\dfrac{\sigma_S^2}{\mu_X^2}$.

It is to be noted that the covariance between X and Y is equal to the covariance between Z^* and Y, that is, $\sigma_{XY} = \sigma_{Z^* Y}$

Now, substituting the value of ρ_{YZ^*} and $C_{Z^*}^2$ in the expressions of bias and MSE, we get:

$$Bias(\hat{\mu}_{RP}) \cong \left(\frac{1-f}{n}\right)\mu_X \left(C_Y^2 - \frac{\rho_{XY}}{\sqrt{1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}}} \sqrt{\left(C_X^2 + W(1-T)\frac{\sigma_S^2}{\mu_X^2}\right)} C_Y \right)$$

$$MSE(\hat{\mu}_{RP}) = \left(\frac{1-f}{n}\right)\mu_X^2 \left(C_X^2 + W(1-T)\frac{\sigma_S^2}{\mu_X^2} + C_Y^2 - 2\frac{\rho_{XY}}{\sqrt{\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)}} \sqrt{\left(C_X^2 + W(1-T)\frac{\sigma_S^2}{\mu_X^2}\right)} C_Y \right)$$

(7)

It is interesting to observe that for $T=0$ and $W=1$, $MSE(\hat{\mu}_{RP})$ coincides with the MSE of ratio estimator by Sousa et al. (2010) and for $T=0,$ the MSE of our proposed ratio estimator is exactly the same as the ratio estimator proposed by Gupta et al. (2014).

4.2 Product Estimator

Based on the data obtained by Gupta et al. (2010) optional RRT, the proposed product estimator is given as:

$$\hat{\mu}_{PP} = \frac{\overline{y}.\overline{z}^*}{\mu_Y},$$

Following the steps discussed in section 4.1, the bias and MSE of the proposed product estimator $\hat{\mu}_{PP}$ are given by

$$Bias(\hat{\mu}_{PP}) = \left(\frac{1-f}{n}\right)\mu_{Z^*}\rho_{YZ^*}C_{Z^*}C_Y,$$

$$MSE(\hat{\mu}_{PP}) = \left(\frac{1-f}{n}\right)\mu_{Z^*}^2\left(C_{Z^*}^2 + C_Y^2 + 2\rho_{YZ^*}C_{Z^*}C_Y\right),$$

Now, replacing the value of $C_{Z^*}^2$ and ρ_{YZ^*} in above expressions, we get the expression of Bias and MSE of $\hat{\mu}_{PP}$ as follows:

$$Bias(\hat{\mu}_{PP}) \cong \left(\frac{1-f}{n}\right)\mu_X \frac{\rho_{XY}}{\sqrt{1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}}}\sqrt{\left(C_X^2 + W(1-T)\frac{\sigma_S^2}{\mu_X^2}\right)}C_y,$$

$$MSE(\hat{\mu}_{PP}) \cong \left(\frac{1-f}{n}\right)\mu_X^2\left(C_X^2 + W(1-T)\frac{\sigma_S^2}{\mu_X^2} + C_Y^2 + 2\frac{\rho_{XY}}{\sqrt{\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)}}\sqrt{\left(C_X^2 + W(1-T)\frac{\sigma_S^2}{\mu_X^2}\right)}C_Y\right). \quad (8)$$

4.3 Regression Estimator

The proposed regression estimator is given by

$$\hat{\mu}_{\mathrm{Re}gP} = \bar{z}^* + \hat{\beta}_{Z^*Y}(\mu_Y - \bar{y}),$$

Where $\hat{\beta}_{Z^*Y}$ is the sample regression coefficient between Z^* and Y. The bias and MSE of the proposed regression estimator $\hat{\mu}_{\mathrm{Re}gP}$ is given by

$$Bias(\hat{\mu}_{\mathrm{Re}gP}) \cong -\beta_{Z^*Y}\left(\frac{1-f}{n}\right)\left\{\frac{\mu_{12}}{\mu_{11}} - \frac{\mu_{03}}{\mu_{02}}\right\},$$

Where $\beta_{Z^*Y} = \beta_{XY}$ and $\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}(Z_i^* - \bar{Z}^*)^r (y_i - \bar{Y})^s$

$$MSE(\hat{\mu}_{\mathrm{Re}gP}) \cong \left(\frac{1-f}{n}\right)\mu_{z^*}^2 C_{z^*}^2 \left\{1-\rho_{YZ^*}^2\right\},$$

Now, replacing the value of $C_{Z^*}^2$ and ρ_{YZ^*} in above expressions, we get following expressions of bias and MSE of $\hat{\mu}_{\mathrm{Re}gP}$.

$$Bias(\hat{\mu}_{\mathrm{Re}gP}) \cong -\beta_{XY}\left(\frac{1-f}{n}\right)\left\{\frac{\mu_{12}}{\mu_{11}} - \frac{\mu_{03}}{\mu_{02}}\right\},$$

$$MSE(\hat{\mu}_{\mathrm{Re}gP}) \cong \left(\frac{1-f}{n}\right)\mu_X^2\left(C_X^2 + W(1-T)\frac{\sigma_S^2}{\mu_X^2}\right)\left\{1 - \frac{\rho_{XY}^2}{1+W(1-T)\dfrac{\sigma_S^2}{\sigma_X^2}}\right\},$$

$$MSE(\hat{\mu}_{\mathrm{Re}gP}) \cong \left(\frac{1-f}{n}\right)\sigma_X^2\left\{\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right) - \rho_{XY}^2\right\}. \quad (9)$$

Again, note that for $W=1$ and $T=0$, the MSE of the proposed regression estimator coincides with the MSE of Gupta et al. (2012) proposed regression estimator and for $T=0$, the MSE of our proposed regression estimator is same as the MSE of Gupta et al. (2014) proposed estimator.

5. Algebraic and Numerical Comparisons

In this section, we compare proposed ratio, product and regression estimators with Sousa et al. (2010), Gupta et al. (2012) and Gupta et al. (2014) estimators both algebraically and numerically, and establish the superiority of our proposed ratio, product and regression estimators. For comparing different estimators, we consider the Percent Relative Efficiency (PRE*)* as the performance criterion.

5.1 Relative efficiency comparisons of different Ratio Estimators

(i) The proposed ratio estimator will be more efficient than the Sousa et al. (2010) ratio estimator if

$$MSE(\hat{\mu}_{RP}) < MSE(\hat{\mu}_{RS})$$

$$(1-W(1-T))\frac{\sigma_s^2}{\mu_x^2} - 2\rho_{XY}C_Y\left(\frac{\sqrt{\left(C_X^2+\frac{\sigma_s^2}{\mu_x^2}\right)}}{\sqrt{\left(1+\frac{\sigma_s^2}{\sigma_x^2}\right)}} - \frac{\sqrt{\left(C_X^2+W(1-T)\frac{\sigma_s^2}{\mu_x^2}\right)}}{\sqrt{\left(1+W(1-T)\frac{\sigma_s^2}{\sigma_x^2}\right)}}\right) > 0$$

$$(1-W(1-T))\frac{\sigma_s^2}{\mu_x^2}\sqrt{\left(1+\frac{\sigma_s^2}{\sigma_x^2}\right)\left(1+W(1-T)\frac{\sigma_s^2}{\sigma_x^2}\right)} - 2\rho_{XY}C_Y\left(\sqrt{\left(C_X^2+\frac{\sigma_s^2}{\mu_x^2}\right)\left(1+W(1-T)\frac{\sigma_s^2}{\sigma_x^2}\right)} - \sqrt{\left(C_X^2+W(1-T)\frac{\sigma_s^2}{\mu_x^2}\right)\left(1+\frac{\sigma_s^2}{\sigma_x^2}\right)}\right) > 0$$

Now, consider only the second term

$$\sqrt{\left(C_X^2+\frac{\sigma_s^2}{\mu_x^2}\right)\left(1+W(1-T)\frac{\sigma_s^2}{\sigma_x^2}\right)} - \sqrt{\left(C_X^2+W(1-T)\frac{\sigma_s^2}{\mu_x^2}\right)\left(1+\frac{\sigma_s^2}{\sigma_x^2}\right)}$$

$$= \sqrt{\left(C_X^2+\frac{\sigma_s^2}{\mu_x^2}+W(1-T)\left(C_X^2+\frac{\sigma_s^2}{\mu_x^2}\right)\frac{\sigma_s^2}{\sigma_x^2}\right)} - \sqrt{\left(C_X^2+W(1-T)\frac{\sigma_s^2}{\mu_x^2}+\left(C_X^2+W(1-T)\frac{\sigma_s^2}{\mu_x^2}\right)\frac{\sigma_s^2}{\sigma_x^2}\right)}$$

$$= \sqrt{\left(C_X^2+\frac{\sigma_s^2}{\mu_x^2}+W(1-T)C_X^2\frac{\sigma_s^2}{\sigma_x^2}+W(1-T)\frac{\left(\sigma_s^2\right)^2}{\mu_x^2\sigma_x^2}\right)} - \sqrt{\left(C_X^2+W(1-T)\frac{\sigma_s^2}{\mu_x^2}+C_X^2\frac{\sigma_s^2}{\sigma_x^2}+W(1-T)\frac{\left(\sigma_s^2\right)^2}{\mu_x^2\sigma_x^2}\right)}$$

$$= 0$$

Thus, the above inequality becomes

$$(1-W(1-T))\frac{\sigma_s^2}{\sigma_x^2}\sqrt{\left(1+\frac{\sigma_s^2}{\sigma_x^2}\right)\left(1+W(1-T)\frac{\sigma_s^2}{\sigma_x^2}\right)}-2\rho_{XY}C_Y\left(0\right)>0$$

$$(1-W(1-T))\frac{\sigma_s^2}{\sigma_x^2}\sqrt{\left(1+\frac{\sigma_s^2}{\sigma_x^2}\right)\left(1+W(1-T)\frac{\sigma_s^2}{\sigma_x^2}\right)}>0$$

The above expression is always true because all the quantities involved in the expression on right hand side are positive.

(ii) The proposed ratio estimator of the population mean performs better than Gupta et al. (2014) estimator if

$$MSE(\hat{\mu}_{RP})<MSE(\hat{\mu}_{RWG})$$

$$WT\frac{\sigma_s^2}{\mu_x^2}-2\rho_{XY}C_Y\left(\frac{\sqrt{\left(C_X^2+W\frac{\sigma_s^2}{\mu_x^2}\right)}}{\sqrt{\left(1+W\frac{\sigma_s^2}{\sigma_x^2}\right)}}-\frac{\sqrt{\left(C_X^2+W(1-T)\frac{\sigma_s^2}{\mu_x^2}\right)}}{\sqrt{\left(1+W(1-T)\frac{\sigma_s^2}{\sigma_x^2}\right)}}\right)>0$$

$$WT\frac{\sigma_S^2}{\mu_X^2}\sqrt{\left(1+W\frac{\sigma_S^2}{\sigma_X^2}\right)\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)}-2\rho_{XY}C_Y\left(\sqrt{\left(C_X^2+W\frac{\sigma_s^2}{\mu_x^2}\right)\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)}-\sqrt{\left(C_X^2+W(1-T)\frac{\sigma_S^2}{\mu_X^2}\right)\left(1+W\frac{\sigma_S^2}{\sigma_X^2}\right)}\right)>0$$

Now, consider the second term only

$$\sqrt{\left(C_X^2+W\frac{\sigma_s^2}{\mu_x^2}\right)\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)}-\sqrt{\left(C_X^2+W(1-T)\frac{\sigma_S^2}{\mu_X^2}\right)\left(1+W\frac{\sigma_S^2}{\sigma_X^2}\right)}$$

$$=\sqrt{\left(C_X^2+W\frac{\sigma_s^2}{\mu_x^2}+W(1-T)\left(C_X^2+W\frac{\sigma_s^2}{\mu_x^2}\right)\frac{\sigma_S^2}{\sigma_X^2}\right)}-\sqrt{\left(C_X^2+W(1-T)\frac{\sigma_S^2}{\mu_X^2}+W\left(C_X^2+W(1-T)\frac{\sigma_S^2}{\mu_X^2}\right)\frac{\sigma_S^2}{\sigma_X^2}\right)}$$

$$= \sqrt{\left(C_X^2 + W\frac{\sigma_s^2}{\mu_x^2} + W(1-T)C_X^2\frac{\sigma_S^2}{\sigma_X^2} + W^2(1-T)\frac{\left(\sigma_S^2\right)^2}{\mu_x^2\sigma_X^2}\right)} - \sqrt{\left(C_X^2 + W(1-T)\frac{\sigma_S^2}{\mu_X^2} + WC_X^2\frac{\sigma_S^2}{\sigma_X^2} + W^2(1-T)\frac{\left(\sigma_S^2\right)^2}{\mu_X^2\sigma_X^2}\right)}$$

$$= 0$$

The above inequality becomes

$$WT\frac{\sigma_S^2}{\mu_X^2}\sqrt{\left(1+W\frac{\sigma_S^2}{\sigma_X^2}\right)\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)} - 2\rho_{XY}C_Y\left(0\right) > 0$$

$$WT\frac{\sigma_S^2}{\mu_X^2}\sqrt{\left(1+W\frac{\sigma_S^2}{\sigma_X^2}\right)\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)} > 0$$

The above expression is always true because all the terms involved on right hand side are positive.

5.2 Relative efficiency comparison of different Product estimators

(i)The proposed product estimator performed better than Gupta et al. (2014) suggested product estimator if

$$MSE(\hat{\mu}_{PP}) < MSE(\hat{\mu}_{PWG})$$

$$WT\frac{\sigma_s^2}{\mu_x^2} + 2\rho_{XY}C_Y\left(\frac{\sqrt{\left(C_X^2 + W\frac{\sigma_s^2}{\mu_x^2}\right)}}{\sqrt{\left(1+W\frac{\sigma_s^2}{\sigma_x^2}\right)}} - \frac{\sqrt{\left(C_X^2 + W(1-T)\frac{\sigma_s^2}{\mu_x^2}\right)}}{\sqrt{\left(1+W(1-T)\frac{\sigma_s^2}{\sigma_x^2}\right)}}\right) > 0$$

$$WT\frac{\sigma_S^2}{\mu_X^2}\sqrt{\left(1+W\frac{\sigma_S^2}{\sigma_X^2}\right)\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)} + 2\rho_{XY}C_Y\left(\sqrt{\left(C_X^2 + W\frac{\sigma_s^2}{\mu_x^2}\right)\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)} - \sqrt{\left(C_X^2 + W(1-T)\frac{\sigma_S^2}{\mu_X^2}\right)\left(1+W\frac{\sigma_S^2}{\sigma_X^2}\right)}\right) > 0$$

As we have already proved that

$$\sqrt{\left(C_X^2+W\frac{\sigma_s^2}{\mu_x^2}\right)\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)}-\sqrt{\left(C_X^2+W(1-T)\frac{\sigma_S^2}{\mu_X^2}\right)\left(1+W\frac{\sigma_S^2}{\sigma_X^2}\right)}=0,$$

we have

$$WT\frac{\sigma_S^2}{\mu_X^2}\sqrt{\left(1+W\frac{\sigma_S^2}{\sigma_X^2}\right)\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)}-2\rho_{XY}C_Y(0)>0$$

$$WT\frac{\sigma_S^2}{\mu_X^2}\sqrt{\left(1+W\frac{\sigma_S^2}{\sigma_X^2}\right)\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)}>0.$$

The above expression is always true because all the involved terms are positive.

5.3. **Relative efficiency comparison of different Regression estimators**

(i) The proposed regression estimator performs better than usual regression estimators if

$$MSE(\hat{\mu}_{\mathrm{Re}gP})<MSE(\hat{\mu}_{XP})$$

$$\left(\frac{1-f}{n}\right)\sigma_X^2\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}-\rho_{XY}^2\right)<\left(\frac{1-f}{n}\right)\sigma_X^2\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}\right)$$

$$\rho_{XY}^2>0$$

Now we observe from above expressions, $MSE(\hat{\mu}_{\mathrm{Re}gP})$ is always less than $MSE(\hat{\mu}_{XP})$ because $\rho_{XY}^2>0$, which is always true.

(ii)The proposed regression estimator performs better than Gupta et al. (2012) regression estimator if

$$MSE(\hat{\mu}_{\mathrm{Re}gP})<MSE(\hat{\mu}_{\mathrm{Re}gG})$$

$$\left(\frac{1-f}{n}\right)\sigma_X^2\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}-\rho_{XY}^2\right)<\left(\frac{1-f}{n}\right)\sigma_X^2\left(1+\frac{\sigma_S^2}{\sigma_X^2}-\rho_{XY}^2\right)$$

$$1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}-\rho_{XY}^2<1+\frac{\sigma_S^2}{\sigma_X^2}-\rho_{XY}^2$$

$$W(1-T)\frac{\sigma_S^2}{\sigma_X^2}<\frac{\sigma_S^2}{\sigma_X^2}$$

$$W(1-T)<1.$$

The above expression $W(1-T)<1$ is always true because both T and W lie between 0 and 1$(0<T<1,\ 0\leq W<1)$.

(iii) The proposed regression estimator performs better than Gupta et al. (2014) regression estimator if

$$MSE(\hat{\mu}_{\mathrm{Re}\,gP})<MSE(\hat{\mu}_{\mathrm{Re}\,gWG})$$

$$\left(\frac{1-f}{n}\right)\sigma_X^2\left(1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}-\rho_{XY}^2\right)<\left(\frac{1-f}{n}\right)\sigma_X^2\left(1+W\frac{\sigma_S^2}{\sigma_X^2}-\rho_{XY}^2\right)$$

$$1+W(1-T)\frac{\sigma_S^2}{\sigma_X^2}-\rho_{XY}^2<1+W\frac{\sigma_S^2}{\sigma_X^2}-\rho_{XY}^2$$

$$W(1-T)\frac{\sigma_S^2}{\sigma_X^2}<W\frac{\sigma_S^2}{\sigma_X^2}$$

$$1-T<1$$

$$T>0,$$

Which is always true because T always lies between 0 and 1

5.4 Numerical Comparisons

To know the extent of the *PRE* of different proposed estimator, we calculated the *PRE* numerically. The values of the parameters are fixed same as considered by Gupta et al. (2014). The *PREs* of the different estimators are defined as

$$PRE=\frac{MSE(\hat{\mu}_{XS})}{MSE(\hat{\mu}_i)}\times 100$$

Where $i = XS, RS, \operatorname{Re}gG, RWG, \operatorname{Re}gWG, XP, RP, \operatorname{Re}gP.$

We considered $N = 5000$, $n = 200$, $\rho_{XY} = 0.8$, $\mu_X = 6$, $\mu_Y = 4$, $\sigma_X = 3$, $\sigma_S = 3$, $\sigma_Y = 2$, $W = 0.1, 0.2, ..., 1.0$ and $T = 0.1, 0.3, 0.5, 0.7, 0.9$ and calculated *PRE* of each estimator as defined above. The PRE results, so obtained, are presented in the Table 1, below.

Table 1: *PRE* comparison of different estimators when $N = 5000$, $n = 200$, $\rho_{XY} = 0.8$, $\mu_X = 6$, $\mu_Y = 4$, $\sigma_X = 3$, $\sigma_S = 3$

	$\hat{\mu}_{XS}$	$\hat{\mu}_{RS}$	$\hat{\mu}_{RegG}$	$\hat{\mu}_{RWG}$	$\hat{\mu}_{RegWG}$	$\hat{\mu}_{XP}$	$\hat{\mu}_{RP}$	$\hat{\mu}_{RegP}$
W	$T = 0.1$							
0.1	100	142.85	147.06	400	434.78	183.48	408.16	444.44
0.2	100	142.85	147.06	333.33	357.14	169.49	344.82	370.37
0.3	100	142.85	147.06	285.71	303.03	157.48	298.50	317.46
0.4	100	142.85	147.06	250	263.15	147.05	263.15	277.77
0.5	100	142.85	147.06	222.22	232.55	137.93	235.29	246.91
0.6	100	142.85	147.06	200	208.33	129.87	212.76	222.22
0.7	100	142.85	147.06	181.81	188.67	122.69	194.17	202.02
0.8	100	142.85	147.06	166.66	172.41	116.27	178.57	185.18
0.9	100	142.85	147.06	153.84	158.73	110.49	165.28	170.94
1	100	142.85	147.06	142.85	147.05	105.26	153.84	158.73
	$T = 0.3$							
0.1	100	142.85	147.06	400	434.78	186.91	425.53	465.11
0.2	100	142.85	147.06	333.33	357.14	175.44	370.37	400
0.3	100	142.85	147.06	285.71	303.03	165.29	327.87	350.88
0.4	100	142.85	147.06	250	263.15	156.25	294.12	312.5
0.5	100	142.85	147.06	222.22	232.55	148.15	266.67	281.69
0.6	100	142.85	147.06	200	208.33	140.85	243.90	256.41
0.7	100	142.85	147.06	181.81	188.67	134.23	224.72	235.29
0.8	100	142.85	147.06	166.66	172.41	128.21	208.33	217.39
0.9	100	142.85	147.06	153.84	158.73	122.69	194.17	202.02
1	100	142.85	147.06	142.85	147.05	117.64	153.84	158.73
	$T = 0.5$							
0.1	100	142.85	147.06	400	434.78	190.48	444.44	487.81
0.2	100	142.85	147.06	333.33	357.14	181.82	400	434.78
0.3	100	142.85	147.06	285.71	303.03	173.91	363.64	392.16
0.4	100	142.85	147.06	250	263.15	166.67	333.3	357.14

0.5	100	142.85	147.06	222.22	232.55	160	307.69	327.87
0.6	100	142.85	147.06	200	208.33	153.85	285.71	303.03
0.7	100	142.85	147.06	181.81	188.67	148.15	266.67	281.69
0.8	100	142.85	147.06	166.66	172.41	142.86	250	263.16
0.9	100	142.85	147.06	153.84	158.73	137.93	235.29	246.91
1	100	142.85	147.06	142.85	147.05	133.33	222.22	232.56
	$T = 0.7$							
0.1	100	142.85	147.06	400	434.78	194.17	465.12	512.82
0.2	100	142.85	147.06	333.33	357.14	188.68	434.78	476.19
0.3	100	142.85	147.06	285.71	303.03	183.49	408.16	444.44
0.4	100	142.85	147.06	250	263.15	178.57	384.62	416.67
0.5	100	142.85	147.06	222.22	232.55	173.91	363.64	392.17
0.6	100	142.85	147.06	200	208.33	169.49	344.83	370.37
0.7	100	142.85	147.06	181.81	188.67	165.29	327.87	350.88
0.8	100	142.85	147.06	166.66	172.41	161.29	312.5	333.33
0.9	100	142.85	147.06	153.84	158.73	157.48	298.51	317.46
1	100	142.85	147.06	142.85	147.05	153.84	285.71	303.03
	$T = 0.9$							
0.1	100	142.85	147.06	400	434.78	198.02	487.80	540.54
0.2	100	142.85	147.06	333.33	357.14	196.08	476.19	526.32
0.3	100	142.85	147.06	285.71	303.03	194.17	465.12	512.82
0.4	100	142.85	147.06	250	263.15	192.31	454.55	500
0.5	100	142.85	147.06	222.22	232.55	190.48	444.44	487.80
0.6	100	142.85	147.06	200	208.33	188.68	434.78	476.19
0.7	100	142.85	147.06	181.81	188.67	186.92	425.53	465.11
0.8	100	142.85	147.06	166.66	172.41	185.19	416.67	454.54
0.9	100	142.85	147.06	153.84	158.73	183.48	408.16	444.44
1	100	142.85	147.06	142.85	147.05	181.81	400	434.78

From Table 1, it is observed that *PRE* of the proposed ratio estimator is greater than 100 for all values of $T(0 < T < 1)$ which proves the superiority of our proposed estimator over usual mean per unit estimator of Sousa et al. (2010). The *PRE* of the proposed ratio estimator is greater than *PRE* of Sousa et al. (2010) ratio estimator and Gupta et al. (2014) ratio estimator for all values of $T(0 < T < 1)$ and $\rho_{XY}^2 > 0$. From Tables 1-5, it can also be observed that as the value of T increases the *PRE* of the proposed ratio estimator increases. The *PRE* of the proposed regression estimator is greater than 100 for all values of T and $\rho_{XY}^2 > 0$. Thus, the proposed

Regression estimator proves to be superior than Sousa et al. (2010). The *PRE* of the proposed regression estimator is greater than Gupta et al. (2012) regression estimator and Gupta et al. (2014) regression estimator for all values of $T(0<T<1)$ and $\rho_{XY}^2>0$. As expected, when the value of T increases, the efficiency of our proposed regression estimator also increases.

Conclusion

The use of auxiliary information is proved to be very useful in enhancing the efficiency of the estimators. Sousa et al. (2010) used a regression type estimator of finite population mean which is based on an additive scrambled response model. Gupta et al. (2012) proposed a regression type estimator which showed a significant improvement in the efficiency of the estimator. Gupta et al. (2014) proposed an optional RRT. They used a ratio and regression estimators for improving efficiency of the estimates and showed the superiority of their results over Sousa et al. (2010) and Gupta et al. (2012). We used a two-stage ORR model and proposed ratio, product and regression type estimator utilizing non-sensitive auxiliary information. Our proposed ratio, product and regression estimators perform better than Sousa et al. (2010), Gupta et al. (2012) and Gupta et al. (2014) estimators.

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