DOI: http://doi.org/10.4038/sljastats.v22i3.8052

Sri Lankan Journal of Applied Statistics, Vol (22-3)

Estimation of Sensitive Variable in Two-Phase Sampling Under Measurement Error And Non-Response Using ORRT Models

S. Kumar[1] and S. P. Kour[2*]

[1,2]Department of Statistics, University of Jammu, J&K, India

*Corresponding author: sanamkour903@gmail.com

Received: 23rd October 2021 / Revised: 19th December 2021 / Published: 31st December 2021

ABSTRACT

In human surveys, people are asked highly confidential questions concerning a sensitive variable. This article concentrate on the estimation of population mean of sensitive variable under two phase sampling using ORRT models. Also, the presence of non response and measurement errors is of concern while discussing the properties of the proposed estimators. The conditions under which the proposed estimator perform relatively better than the estimators based on recent studies are obtained. Simulation study also carried out in different situations over the data set of natural population and fictitious population to support the theoretical findings.

Keywords: Scrambling Variable(s), Bias, Mean square error (MSE), Percent Relative Efficiency (PRE), Unified Measure.

1 Introduction

In sampling literature, a diverse range of techniques are present which focus on the ways of utilizing auxiliary information for accomplishing better and efficient results. An auxiliary information is the information which is often available in every unit of the population. For example, number of beds available in different hospitals is known in health care surveys. If such information lacks, it is sometimes relatively inexpensive to take large preliminary sample where auxiliary variable measured alone. Such practice is applicable on two-phase (or double) sampling in which we obtain the information about auxiliary variable(s) from a larger sample at first phase and comparatively small sample from the second phase. Neyman (1938) were the first who initiated the concept of two-phase sampling for estimating the population parameters. Subsequently, many authors such as Sukhatme (1962), Singh and Vishwakarma (2007), Sanaullah et al. (2014), Zaman and Kadilar (2019), Misra et al. (2021) etc have worked on two-phase sampling.

It is common pattern in survey sampling that information on sensitive variable would be collected by using randomized response technique (RRT) which was introduced by Warner (1965) because direct reliable information on variable of interest is sometimes may not possible. Eichhorn and Hayre (1983) introduced a multiplicative RRT model. Every respondent in the preceding models is obliged to offer a scrambled response. Moreover, researchers have discovered that a question may be sensitive for one respondent but not for another. So, Gupta et al. (2002) modified Eichhorn and Hayre's (1983) multiplicative scrambling RRT model and developed an Optional randomized response technique (ORRT), which allows researchers to estimate not only the mean of the variable of interest, but also the sensitivity level P. Furthermore, Gupta et al. (2014; 2018; 2012), Zhang et al. (2018), Mushtaq and Amin (2020) etc have worked on estimation of mean of sensitive variable using ORRT.

In statistical studies, the problem of non-response is unavoidable. Hansen and Hurwitz (1946) were the first who introduced the estimation method to deal with the problem of non-response. They evoke a procedure of taking sub-sample of non-respondents after the mail attempt and then obtain information from the sub-sample by personal interview. Further, the technique was studied by Cochran (1977), Kumar and Bhougal (2011), Guha and Chandra (2019) under two-phase sampling plan. Diana et al. (2014) have designed the problem of non-response in case of sensitive variable. Equivalent to non-response, the investigators also confront the problem of measurement error while collecting the information from the respondents. It is the deviation between the value that is determined and the actual value of a variable. Many researchers like Cochran (1968), Singh and Karpe (2010), Singh et al. (2014) etc have studied the measurement error while utilizing the auxiliary information. Khalil et al. (2018; 2021) have estimated the mean of the sensitive variable in the presence of measurement error. Furthermore, Kumar and Bhougal (2018) have studied the problem of non-response and measurement error simultaneously and Zahid and Shabbir (2019), Zhang et al. (2021) etc have studied the mean estimation of sensitive variable under measurement error and non-response together.

Taking inspiration from previous researches, we propose a regression-cum-exponential estimator for the estimation of sensitive variable using ORRT models in two-phase sampling and studied their properties. The scrambling technique is described in section 2. In section 3, some existing estimators in terms of estimation in presence of sensitive variable using ORRT model in two-phase sampling have been discussed. In section 4, an estimator in two-phase sampling is proposed and studied its properties are discussed with some special cases. Theoretical and numerical comparisons are made and conditions are obtained under which our proposed estimator outperforms other current estimators in section 5 and 6. Finally, in section 7, there are some concluding thoughts.

2 Scrambling Technique

In this section, let us consider a finite population having distinctive units $\Upsilon = \Upsilon_1, \Upsilon_2$, ..., Υ_N in which Y be the sensitive study variable, X be non-sensitive auxiliary variable which is positively correlated with Y. Let T and S be two scrambling variables with known variances and we take T with a mean (μ_T) of 1 and S with a mean (μ_s) of 0. Let P be the probability that the respondent finds the question sensitive. Let n' is the number of units in the first phase sample units whereas n is the number of units in the second phase sample units. Only in the second phase sample units both sensitive study and non-sensitive auxiliary variable are ascertained, in the first phase sample units only non-sensitive auxiliary variable is observed because sensitive study variable is expensive in nature to measure. The two-phase sampling strategy is given below

1. In the first phase, a large sample of a fixed size n' is drawn from N to observe only X or non-sensitive auxiliary variable.

2. In the second phase sample, a sub-sample of fixed size n is drawn from n' to observe Y and X , so that ($n < n'$).

Using the privacy protection measure $\Omega = E(Z - Y)^2$ given by Yan et al. (2008), we can easily calculate the privacy levels of the Pollock and Beck (1976) model and the Diana and Perri (2011) model. These are given by

$$\Omega_{PB} = \sigma_s^2$$

and

$$\Omega_{DP} = \sigma_T^2(\mu_y^2 + \sigma_y^2) + \sigma_s^2$$

A simple additive RRT model is utilized Gupta et al. (2012), where the scrambling response is given by $Y + S$. Then, a more general RRT model is used by Diana and Perri (2011) where the scrambling response is given by $TY + S$. Further, the simple additive model is more effcient but the general model has greater privacy is demonstrated by Khailil et al. (2018). Moreover, Gupta et al. (2018) investigates that the general RRT model is better when we use a unified measure of model quality i.e. $\delta = \frac{Var(Z)}{\Omega}$, where Z is the scrambled response. It may be determined that

$$\delta_{additiveRRT} = 1 + \frac{S_y^2}{S_S^2} > 1 + \frac{S_y^2}{S_S^2 + S_T^2(\bar{y}^2 + S_y^2)} = \delta_{generalRRT} \tag{1}$$

Since ORRT model is more efficient and reliable than RRT, we add optionally to the Diana and Perri (2011) model. In the ORRT version, the respondent may answer in the two ways given in (2) depending on whether the respondent considers the question sensitive or not. Therefore, when we apply ORRT in the second phase, then the reported response of the respondents is

$$Z = \begin{cases} Y & \text{with probability 1-P} \\ TY + S & \text{with probability P,} \end{cases} \tag{2}$$

where it is assumed that $\mu_T = E(T) = 1$ and $\mu_s = E(S) = 0$
The randomized linear model is written as $Z = (TY + S)Q + Y(1 - P)$, where Q $\sim$ Bernoulli(P) with $E(Q) = P$, $Var(Q) = P(1 - P)$ and $E(Q^2) = Var(Q) + E^2(Q) = P$. And the mean and variance of (2) are given by $E(Z) = (\mu_T P + 1 - P)Y + \mu_s P$ and $Var(Z) = (Y^2\sigma_T^2 + \sigma_s^2)P$.

Let $\hat{y}_i$ be a transformation of the randomized response on the i^{th} unit whose expectation under the randomization mechanics is the true response y_i and is given by

$$\hat{y}_i = \frac{z_i - \mu_s}{\mu_T P + 1 - P} \tag{3}$$

with $E_R(\hat{y}_i) = y_i$ and $Var_R(\hat{y}_i) = \frac{(y_i^2\sigma_T^2 + \sigma_s^2)P}{(\mu_T P + 1 - P)^2} = \tau_i$.

2.1 Modified Hansen and Hurwitz Technique

Hansen and Hurwitz (1946) considered a method in order to deal with this problem and introduced a new technique of sub-sampling the non-respondents. They proposed a double sampling scheme for estimating the population mean using mail or phone survey at the first attempt and then apply face-to-face interview at the second attempt to receive more information. If the variable of interest is sensitive it may cause non-response bias. The respondent may provide mendacious response in the face-to-face interview. In order to encourage the respondents to answer a sensitive survey question truthfully, we give respondents an opportunity to scramble the response using ORRT model given in (2) in the second phase of Hansen and Hurwitz procedure when there is a face-to-face interview.

The Hansen and Hurwitz (1946) estimator with ORRT model in (2) is given by

$$\hat{\bar{y}} = w_1\bar{y}_1 + w_2\hat{\bar{y}}_2 \tag{4}$$

where $\hat{\bar{y}}_2 = \sum_{i=1}^{n_s}(\frac{\hat{y}_i}{n_s})$ with expectation and variance in the i^{th} phase (i=1,2) under the two-phase sampling are given by

$$E(\hat{\bar{y}}) = \mu_y \tag{5}$$

and

$$Var(\hat{\bar{y}}) = \theta\sigma_y^2 + \theta^*\sigma_{y(2)}^2 + \frac{W_2 f}{n}\left[\frac{\{(\sigma_{y(2)}^2 + \mu_{y(2)}^2)\sigma_T^2 + \sigma_s^2\}P}{(\mu_T P + 1 - P)^2}\right] \tag{6}$$

where $\theta = \frac{(N-n)}{Nn}$ and $\theta^* = \frac{(f-1)W_2}{n}$ and
it is easy to notice that $\frac{W_2 f}{n}\left[\frac{\{(\sigma_{y(2)}^2+\mu_{y(2)}^2)\sigma_T^2+\sigma_s^2\}P}{(\mu_T P+1-P)^2}\right]$ is the penalisation for using ORRT model.

Let (x_i, y_i, z_i) be the ascertained values and (X_i, Y_i, Z_i) be the true values on the variables (X, Y, Z) respectively. Then the measurement errors associated with the study and auxiliary variable (X, Y) in the population and the scrambled variable (Z) in the face-to-face phase be given by $V_i = x_i - X_i$, $U_i = y_i - Y_i$ and $W_i = z_i - Z_i$. These measurement errors are assumed to be random and uncorrelated with mean zero and variances $\sigma_v^2, \sigma_u^2, \sigma_w^2$, respectively.

$$\omega_y = \sum_{i=1}^{n}(y_i - \mu_y), \tag{7}$$

$$\omega_{x'} = \sum_{i=1}^{n}(x_i' - \mu_x), \tag{8}$$

$$\omega_x = \sum_{i=1}^{n}(x_i - \mu_x), \tag{9}$$

$$\omega_u = \sum_{i=1}^{n_1} U_i + \sum_{i-1}^{n_2} W_i \tag{10}$$

and

$$\omega_v = \sum_{i=1}^{n} V_i \tag{11}$$

where U_i, V_i, W_i are measurement errors on Y, X and Z, respectively.

Let $\epsilon_0^* = \frac{1}{n\mu_y}(\omega_y + \omega_u)$, $\epsilon_1^* = \frac{1}{n\mu_x}(\omega_x + \omega_v)$ and $\epsilon_1' = \frac{1}{n\mu_x}\omega_{x'}$.

In other words, $\hat{\bar{y}}^* = (1+\epsilon_0^*)\mu_y$, $\bar{x}^* = (1+\epsilon_1^*)\mu_x$ and $\bar{x}' = (1+\epsilon_1')\mu_x$.

Under the assumption of bivariate normality (Sukhatme et al. (1970)), we have

$$E(\epsilon_0^*) = E(\epsilon_1^*) = E(\epsilon_1') = 0; \tag{12}$$

$$E(\epsilon_0^{*2}) = \frac{1}{\mu_y^2}\left\{\theta(\sigma_y^2 + \sigma_u^2) + \theta^*(\sigma_{y(2)}^2 + \sigma_w^2) + \frac{W_2 f}{n}\left[\frac{\{(\sigma_{y(2)}^2 + \mu_{y(2)}^2)\sigma_T^2 + \sigma_s^2\}P}{(\mu_T P + 1 - P)^2}\right]\right\}; \tag{13}$$

$$E(\epsilon_1^{*2}) = \frac{1}{\mu_x^2}\{\theta(\sigma_x^2 + \sigma_v^2) + \theta^*(\sigma_{x(2)}^2 + \sigma_v^2)\}; \tag{14}$$

$$E(\epsilon_1'^2) = \frac{1}{\mu_x^2}\theta'\sigma_x^2; \tag{15}$$

$$E(\epsilon_0^*\epsilon_1^*) = \theta\rho_{yx}\frac{\sigma_y}{\mu_y}\frac{\sigma_x}{\mu_x} + \theta^*\rho_{zx(2)}\frac{\sigma_z}{\mu_y}\frac{\sigma_{x(2)}}{\mu_x}; \tag{16}$$

$$E(\epsilon_0^*\epsilon_1') = \theta'\rho_{yx}\frac{\sigma_y}{\mu_y}\frac{\sigma_x}{\mu_x}; \tag{17}$$

and

$$E(\epsilon_1^*\epsilon_1') = \frac{1}{\mu_x^2}\theta'\sigma_x^2; \tag{18}$$

where $\theta = \frac{(N-n)}{Nn}$, $\theta^* = \frac{(f-1)W_2}{n}$, $\theta' = \frac{(N-n')}{Nn'}$, $\sigma_z^2 = \sigma_y^2 + \sigma_s^2 P + \sigma_T^2(\sigma_y^2 + \mu_y^2)P$; and

$$\rho_{zx(2)} = \frac{\rho_{yx(2)}}{\sqrt{1 + \frac{\sigma_s^2 P}{\sigma_{y(2)}^2} + \frac{\sigma_T^2(\sigma_{y(2)}^2 + \mu_{y(2)}^2)P}{\sigma_{y(2)}^2}}} \tag{19}$$

3 Some Existing Estimators

Firstly, we introduced some existing estimators in the presence of measurement error and non-response using ORRT model in two-phase sampling.
The usual unbiased estimator is given by

$$\hat{T}_{yd}^{HH} = \hat{\bar{y}}^{*} = w_1\bar{y}_1^{*} + w_2\hat{\bar{y}}_2^{*} \tag{20}$$

The $MSE(\hat{T}_{yd}^{HH})$ upto the second order approximation is given by

$$MSE(\hat{T}_{yd}^{HH}) = \theta(\sigma_y^2 + \sigma_u^2) + \theta^{*}(\sigma_{y(2)}^2 + \sigma_p^2) + \kappa \tag{21}$$

THe MSE of the usual unbiased estimator without measurement error may be obtained when $\sigma_u^2 = \sigma_w^2 = \sigma_v^2 = 0$ in (21) then we get

$$MSE(\hat{T}_{yd}^{HH}) = \theta\sigma_y^2 + \theta^{*}\sigma_{y(2)}^2 + \kappa \tag{22}$$

where $\kappa = \frac{W_2 f}{n}\left[\frac{\{(\sigma_{y(2)}^2+\mu_{y(2)}^2)\sigma_T^2+\sigma_s^2\}P}{(\mu_T P+1-P)^2}\right]$

A ratio estimator corresponding to Gupta et al. (2014) is given by

$$\hat{T}_{rd}^{HH} = \frac{\hat{\bar{y}}^{*}}{\bar{x}^{*}}\mu_x' = \hat{G}_d^{*}\mu_x' \tag{23}$$

where $\hat{G}_d^{*} = \frac{\hat{\bar{y}}^{*}}{\bar{x}^{*}}$ and $\hat{\bar{y}}^{*} = w_1\bar{y}_1^{*} + w_2\hat{\bar{y}}_2^{*}$.

The $MSE(\hat{T}_{rd}^{HH})$ is given by

$$\begin{aligned} MSE(\hat{T}_{rd}^{HH}) = {} & \theta(\sigma_y^2 + G^2\sigma_x^2 - 2G\rho_{yx}\sigma_y\sigma_x) + \theta^{*}(\sigma_{y(2)}^2 + G^2\sigma_{x(2)}^2 - 2G \\ & \rho_{zx(2)}\sigma_z\sigma_{x(2)}) - \theta'(G^2\sigma_x^2 - 2G\rho_{yx}\sigma_y\sigma_x) + \theta(\sigma_u^2 + G^2\sigma_v^2) \\ & +\theta^{*}(\sigma_w^2 + G^2\sigma_v^2) + \kappa \end{aligned} \tag{24}$$

where $G = \frac{\mu_y}{\mu_x}$.

The MSE of ratio estimator without measurement error may be obtained by putting $\sigma_u^2 = \sigma_p^2 = \sigma_v^2 = 0$ in (24), we get

$$\begin{aligned} MSE(\hat{T}_{rd}^{HH}) = {} & \theta(\sigma_y^2 + G^2\sigma_x^2 - 2G\rho_{yx}\sigma_y\sigma_x) + \theta^{*}(\sigma_{y(2)}^2 + G^2\sigma_{x(2)}^2 - \\ & 2G\rho_{zx(2)}\sigma_z\sigma_{x(2)}) - \theta'(G^2\sigma_x^2 - 2G\rho_{yx}\sigma_y\sigma_x) + \kappa \end{aligned} \tag{25}$$

The Zhang et al. (2021) mean estimator of sensitive variable under measurement error and non-response in two-phase sampling is given by

$$\hat{T}_{zd}^{HH} = (\hat{\bar{y}}^* + k(\bar{x}' - \bar{x}^*))\left(\frac{\bar{D}}{\bar{d}}\right)^v \tag{26}$$

where $\bar{d} = \phi(\alpha\bar{x}^* + \beta) + (1-\phi)(\alpha\bar{x}'_x + \beta)$ and $\bar{D} = \alpha\bar{x}' + \beta$.

The $MSE(\hat{T}_{zd}^{HH})$ is given by

$$\begin{aligned} MSE(\hat{T}_{zd}^{HH}) = \theta[\sigma_y^2 + (k+\phi vR_{zd})^2\sigma_x^2 - 2(k+\phi vR_{zd})\rho_{yx}\sigma_y\sigma_x] + \\ \theta^*[\sigma_{y(2)}^2 + (k+\phi vR_{zd})^2\sigma_{x(2)}^2 - 2(k+\phi vR_{zd})\rho_{zx(2)}\sigma_z\sigma_{x(2)}] - \\ \theta'[(k+\phi vR_{zd})^2\sigma_x^2 - 2(k+\phi vR_{zd})\rho_{yx}\sigma_y\sigma_x] + \\ \theta[\sigma_u^2 + (k+\phi vR_{zd})^2\sigma_v^2] + \theta^*[\sigma_w^2 + (k+\phi vR_{zd})^2\sigma_v^2] + \kappa \end{aligned} \tag{27}$$

The $MSE(\hat{T}_{zd}^{HH})$ is minimum when

$$\phi_{opt} = \frac{\begin{aligned}\theta[\rho_{yx}\sigma_y\sigma_x - k(\sigma_x^2+\sigma_v^2)] + \theta^*[\rho_{zx(2)}\sigma_z\sigma_{x(2)} - k(\sigma_{x(2)}^2 + \sigma_v^2) \\ - \theta'[k\sigma_x^2 - \rho_{yx}\sigma_y\sigma_x]\end{aligned}}{vR_{zs}[\theta(\sigma_x^2+\sigma_v^2) + \theta^*\sigma_{x(2)}^2 + \sigma_v^2 - \theta'\sigma_x^2]} \tag{28}$$

where $R_{zd} = \frac{\alpha\mu_y}{\alpha\mu_x+\beta}$.

The resultant minimum $MSE(\hat{T}_{zd}^{HH})$ is given by

$$\begin{aligned} MSE_{min}(\hat{T}_{zd}^{HH}) = \theta(\sigma_y^2 + H^2\sigma_x^2 - 2H\rho_{yx}\sigma_y\sigma_x) + \theta^*(\sigma_{y(2)}^2 + H^2\sigma_{x(2)}^2 - \\ 2H\rho_{zx(2)}\sigma_z\sigma_{x(2)}) - \theta'(H^2\sigma_x^2 - 2H\rho_{yx}\sigma_y\sigma_x) + \\ \theta(\sigma_u^2 + H^2\sigma_v^2) + \theta^*(\sigma_w^2 + H^2\sigma_v^2) + \kappa \end{aligned} \tag{29}$$

where H=$\frac{\theta\rho_{yx}\sigma_y\sigma_x + \theta^*\rho_{zx(2)}\sigma_z\sigma_{x(2)} - \theta'\rho_{yx}\sigma_y\sigma_x}{\theta(\sigma_x^2+\sigma_v^2) + \theta^*(\sigma_{x(2)}^2+\sigma_v^2) - \theta'\sigma_x^2}$.

The MSE of Zhang et al. (2021) estimator without measurement error by putting $\sigma_u^2 = \sigma_p^2 = \sigma_v^2 = 0$ in (29), we get

$$\begin{aligned} MSE_{min}(\hat{T}_{zd}^{HH}) = \theta(\sigma_y^2 + P^2\sigma_x^2 - 2P\rho_{yx}\sigma_y\sigma_x) + \theta^*(\sigma_{y(2)}^2 + P^2\sigma_{x(2)}^2 - \\ 2P\rho_{zx(2)}\sigma_z\sigma_{x(2)}) - \theta'(P^2\sigma_x^2 - 2P\rho_{yx}\sigma_y\sigma_x) + \kappa \end{aligned} \tag{30}$$

4 Proposed Estimator under ORRT Models

An exponential-type estimators are known to perform efficient than the corresponding considered ratio and product-type estimators in terms of having lower mean square error under certain efficiency conditions. It has a wide range of applications, so motivated by above discussions and Khalil et al. (2021) and Zhang et al. (2021), we propose a regression-cum-exponential estimator under ORRT models for population mean in the presence of non-response and measurement error on both the sensitive study (Y) as well as non-sensitive auxiliary variable (X) in two-phase sampling scheme. We propose the following estimator that includes a wide variety of mean estimators in two-phase sampling is given by

$$\hat{T}_{pd}^{HH} = [\hat{\bar{y}}^* + \lambda(\bar{x}' - \bar{x}^*)] \ exp\left[\frac{\phi\bar{x}' + \beta}{\alpha(\phi\bar{x}^* + \beta) + (1-\alpha)(\phi\bar{x}' + \beta)}\right]^{\eta} \quad (31)$$

where λ and η are suitable constants, α is assumed to be an unknown constant whose value is to be determined from optimally considerations and ϕ ($\phi \neq 0$) and β are assumed to be some known parameters of the auxiliary variable X, such as coefficient of variation (C_x), kurtosis ($\beta_2(x)$), and correlation coefficient (ρ_{yx}) etc. Please note that for different values of ϕ and β, we can obtain various estimators. Also, with $\eta = 1$ we get various ratio estimators and with $\eta = -1$ we get various product estimators.

Now, we have considered the special cases of ($\hat{\tau}_{pi}^{HH}$) as following remarks:

Remark 1:

From $\hat{\tau}_{pd}^{HH}$ for $\eta = 1$, we can take the following ratio estimators

(i) By putting $\lambda = 0$, $\alpha = 1$, we have

$$\hat{\tau}_{111}^{HH} = \hat{\bar{y}}^* \ exp\left[\frac{\phi\bar{x}' + \beta}{\phi\bar{x}^* + \beta}\right]. \quad (32)$$

(ii) By putting $\lambda = 1$, $\alpha = 1$, we have

$$\hat{\tau}_{121}^{HH} = [\hat{\bar{y}}^* + (\bar{x}' - \bar{x}^*)] \ exp\left[\frac{\phi\bar{x}' + \beta}{\phi\bar{x}^* + \beta}\right]. \quad (33)$$

(iii) By putting $\lambda = \beta_{yx}$, $\alpha = 1$, we have

$$\hat{\tau}_{131}^{HH} = [\hat{\bar{y}}^* + \beta_{yx}(\bar{x}' - \bar{x}^*)] \ \ exp\left[\frac{\phi\bar{x}' + \beta}{\phi\bar{x}^* + \beta}\right]. \tag{34}$$

(iv) By putting $\lambda = 0$, $\alpha = \hat{\alpha}_{opt}$, we have

$$\hat{\tau}_{141}^{HH} = \hat{\bar{y}}^* \ \ exp\left[\frac{\phi\bar{x}' + \beta}{\hat{\alpha}_{opt}(\phi\bar{x}^* + \beta) + (1 - \hat{\alpha}_{opt})(\phi\bar{x}' + \beta)}\right]. \tag{35}$$

(v) By putting $\lambda = 1$, $\alpha = \hat{\alpha}_{opt}$, we have

$$\hat{\tau}_{151}^{HH} = [\hat{\bar{y}}^* + (\bar{x}' - \bar{x}^*)] \ \ exp\left[\frac{\phi\bar{x}' + \beta}{\hat{\alpha}_{opt}(\phi\bar{x}^* + \beta) + (1 - \hat{\alpha}_{opt})(\phi\bar{x}' + \beta)}\right]. \tag{36}$$

(vi) By putting $\lambda = \beta_{yx}$, $\alpha = \hat{\alpha}_{opt}$, we have

$$\hat{\tau}_{161}^{HH} = [\hat{\bar{y}}^* + \beta_{yx}(\bar{x}' - \bar{x}^*)] \ \ exp\left[\frac{\phi\bar{x}' + \beta}{\hat{\alpha}_{opt}(\phi\bar{x}^* + \beta) + (1 - \hat{\alpha}_{opt})(\phi\bar{x}' + \beta)}\right]. \tag{37}$$

Remark 2:

From $\hat{\tau}_{pd}^{HH}$ for $\eta = -1$, we can take the following estimators

(i) By putting $\lambda = 0$, $\alpha = 1$, we have

$$\hat{\tau}_{212}^{HH} = \hat{\bar{y}}^* \ \ exp\left[\frac{\phi\bar{x}^* + \beta}{\phi\bar{x}' + \beta}\right]. \tag{38}$$

(ii) By putting $\lambda = 1$, $\alpha = 1$, we have

$$\hat{\tau}_{222}^{HH} = [\hat{\bar{y}}^* + (\bar{x}' - \bar{x}^*)] \ \ exp\left[\frac{\phi\bar{x}^* + \beta}{\phi\bar{x}' + \beta}\right]. \tag{39}$$

(iii) By putting $\lambda = \beta_{yx}$, $\alpha = 1$, we have

$$\hat{\tau}_{232}^{HH} = [\hat{\bar{y}}^* + \beta_{yx}(\bar{x}' - \bar{x}^*)] \ \ exp\left[\frac{\phi\bar{x}^* + \beta}{\phi\bar{x}' + \beta}\right]. \tag{40}$$

(iv) By putting $\lambda = 0$, $\alpha = \hat{\alpha}_{opt}$, we have

$$\hat{\tau}_{242}^{HH} = \hat{\bar{y}}^* \ exp\left[\frac{\hat{\alpha}_{opt}(\phi\bar{x}^* + \beta) + (1-\hat{\alpha}_{opt})(\phi\bar{x}' + \beta)}{\phi\bar{x}' + \beta}\right]. \tag{41}$$

(v) By putting $\lambda = 1$, $\alpha = \hat{\alpha}_{opt}$, we have

$$\hat{\tau}_{252}^{HH} = [\hat{\bar{y}}^* + (\bar{x}' - \bar{x}^*)] \ exp\left[\frac{\hat{\alpha}_{opt}(\phi\bar{x}^* + \beta) + (1-\hat{\alpha}_{opt})(\phi\bar{x}' + \beta)}{\phi\bar{x}' + \beta}\right]. \tag{42}$$

(vi) By putting $\lambda = \beta_{yx}$, $\alpha = \hat{\alpha}_{opt}$, we have

$$\hat{\tau}_{262}^{HH} = [\hat{\bar{y}}^* + \beta_{yx}(\bar{x}' - \bar{x}^*)] \ exp\left[\frac{\hat{\alpha}_{opt}(\phi\bar{x}^* + \beta) + (1-\hat{\alpha}_{opt})(\phi\bar{x}' + \beta)}{\phi\bar{x}' + \beta}\right]. \tag{43}$$

Expanding (31) in terms of ϵ's to the first degree of approximation , we have

$$\hat{T}_{pd}^{HH} = [(1+\epsilon_0^*)\mu_y + \lambda\{(1+\epsilon') - (1+\epsilon_1^*)\mu_x\}] \ exp\left[\frac{\phi(1+\epsilon_1') + \beta}{\alpha(\phi(1+\epsilon_1^*) + \beta) + (1-\alpha)(\phi(1+\epsilon') + \beta)}\right]^{\eta} \tag{44}$$

$$\hat{T}_{pd}^{HH} = (\mu_y + \epsilon_0^*\mu_y - \lambda\mu_x(\epsilon_1' - \epsilon_1^*) \ exp[1 + \psi(\epsilon_1'\epsilon_1^*)]^{-\eta} \tag{45}$$

where $\psi = \frac{\alpha\phi\mu_x}{\phi\mu_x + \beta}$.

$$\hat{T}_{pd}^{HH} = \frac{5\mu_y}{2} + \frac{5\mu_y}{2}\epsilon_0^* - \delta_1\epsilon_1^* + \delta_1\epsilon_1' + \delta_2\epsilon_1^{*2} + \delta_2\epsilon_1'^2 - 2\eta\psi\epsilon_1^*\epsilon_1' - 2\eta\psi\epsilon_0^*\epsilon_1^*\mu_y + 2\eta\psi\epsilon_0^*\epsilon_1'\mu_y \tag{46}$$

where $\delta_1 = \left(2\eta\psi\mu_y + \frac{5\lambda\mu_x}{2}\right)$ and $\delta_2 = \left[\frac{3\eta(\eta+1)}{2}\psi^2\mu_y + 2\eta\lambda\psi\mu_x + \frac{\eta^2}{2}\psi^2\mu_y\right]$.

$$(\hat{T}_{pd}^{HH} - \mu_y) = \frac{3\mu_y}{2} + \frac{5\mu_y}{2}\epsilon_0^* - \delta_1\epsilon_1^* + \delta_1\epsilon_1' + \delta_2\epsilon_1^{*2} + \delta_2\epsilon_1'^2 - 2\eta\psi\epsilon_1^*\epsilon_1' - 2\eta\psi\epsilon_0^*\epsilon_1^*\mu_y + 2\eta\psi\epsilon_0^*\epsilon_1'\mu_y \tag{47}$$

Using (47), the $Bias$ and MSE of $\hat{T}_{pd}^{HH}$ is given by

$$Bias(\hat{T}_{pd}^{HH}) \approxeq \frac{3\mu_y}{2} + \frac{\theta}{\mu_x}\left[\frac{\delta_2}{\mu_x}(\sigma_x^2 + \sigma_v^2) - 2\eta\psi\rho_{yx}\sigma_y\sigma_x\right] + \frac{\theta^*}{\mu_x}\left[\frac{\delta_2}{\mu_x}(\sigma_{x(2)}^2 + \sigma_v^2) - 2\eta\psi\rho_{zx(2)}\sigma_z\sigma_{x(2)}\right] - \frac{\theta'}{\mu_x}\left[\frac{\delta_2}{\mu_x}\sigma_x^2 - 2\eta\psi\rho_{yx}\sigma_y\sigma_x\right] \tag{48}$$

The Bias($\hat{T}_{pd}^{HH}$) without measurement error may be obtained by putting $\sigma_v^2 = 0$ in (48)
and

$$MSE(\hat{T}_{pd}^{HH}) = E(\hat{T}_{pd}^{HH} - \mu_y)^2 = \frac{9}{4}\mu_y^2 + \theta\left(\frac{25}{4}(\sigma_y^2 + \sigma_u^2) + \frac{\delta_1^2}{\mu_x^2}(\sigma_x^2 + \sigma_v^2) - 5\frac{\delta_1}{\mu_x}\rho_{yx}\sigma_y\sigma_x\right) + \theta^*\left(\frac{25}{4}(\sigma_{y(2)}^2 + \sigma_w^2) + \frac{\delta_1^2}{\mu_x^2}(\sigma_{x(2)}^2 + \sigma_v^2) - 5\frac{\delta_1}{\mu_x}\rho_{zx(2)}\sigma_z\sigma_{x(2)}\right) - \theta'\left(\frac{\delta_1^2}{\mu_x^2}\sigma_x^2 - 5\frac{\delta_1}{\mu_x}\rho_{yx}\sigma_y\sigma_x\right) + \frac{25}{4}\kappa \tag{49}$$

where $\kappa = \frac{W_2 f}{n}\left[\frac{\{(\sigma_{y(2)}^2 + \mu_{y(2)}^2)\sigma_T^2 + \sigma_s^2\}P}{(\mu_T P + 1 - P)^2}\right]$.

Differentiating (49) with respect to α and equating it to zero, the optimum value of α are found as

$$\hat{\alpha}_{opt} = \frac{5[\theta\rho_{yx}\sigma_y\sigma_x + \theta^*\rho_{zx(2)}\sigma_z\sigma_{x(2)} - \theta'\rho_{yx}\sigma_y\sigma_x] - \lambda[\theta(\sigma_x^2 + \sigma_v^2) + \theta^*(\sigma_{x(2)}^2 + \sigma_v^2) - \theta'\sigma_x^2]}{4\eta\gamma[\theta(\sigma_x^2 + \sigma_v^2) + \theta^*(\sigma_{x(2)}^2 + \sigma_v^2) - \theta'\sigma_x^2]} \tag{50}$$

where $\gamma = \frac{\phi\mu_y}{\phi\mu_x + \beta}$.
Substituting the optimum value in (49), the minimum $MSE(\hat{T}_{pd}^{HH})$ is given by

$$MSE_{min}(\hat{T}_{pd}^{HH}) = \frac{9}{4}\mu_y^2 + \theta\left(\frac{25}{4}(\sigma_y^2 + \sigma_u^2) + \Delta^2(\sigma_x^2 + \sigma_v^2) - 5\Delta\rho_{yx}\sigma_y\sigma_x\right) + \theta^*\left(\frac{25}{4}(\sigma_{y(2)}^2 + \sigma_w^2) + \Delta^2(\sigma_{x(2)}^2 + \sigma_v^2) - 5\Delta\rho_{zx(2)}\sigma_z\sigma_{x(2)}\right) - \theta'(\Delta\sigma_x^2 - 5\Delta\rho_{yx}\sigma_y\sigma_x) + \frac{25}{4}\kappa \tag{51}$$

where $\Delta = \frac{5[\theta\rho_{yx}\sigma_y\sigma_x+\theta^*\rho_{zx(2)}\sigma_z\sigma_{x(2)}-\theta'\rho_{yx}\sigma_y\sigma_x]-\lambda[\theta(\sigma_x^2+\sigma_v^2)+\theta^*(\sigma_{x(2)}^2+\sigma_v^2)-\theta'\sigma_x^2]}{2[\theta(\sigma_x^2+\sigma_v^2)+\theta^*(\sigma_{x(2)}^2+\sigma_v^2)-\theta'\sigma_x^2]}$.

When we let $\sigma_u^2 = \sigma_w^2 = \sigma_v^2 = 0$ in (51), then the $MSE_{min}(\hat{T}_{pd}^{HH})$ without measurement error which is given by

$$MSE_{min}(\hat{T}_{pd}^{HH}) = \frac{9}{4}\mu_y^2 + \theta\left(\frac{25}{4}\sigma_y^2 + \Delta^2\sigma_x^2 - 5\Delta\rho_{yx}\sigma_y\sigma_x\right) + \theta^*\left(\frac{25}{4}\sigma_{y(2)}^2 + \Delta^2\sigma_{x(2)}^2 - 5\Delta\rho_{zx(2)}\sigma_z\sigma_{x(2)}\right) - \theta'(\Delta\sigma_x^2 - 5\Delta\rho_{yx}\sigma_y\sigma_x) + \frac{25}{4}\kappa \tag{52}$$

By using (52) for different values of λ and α, we get the minimum MSE of $\hat{T}_{pd}^{HH}$.

5 Efficiency Comparisons

In this section, we find the conditions by comparing the MSE expressions of the proposed estimator '$\hat{T}_{pd}^{HH}$'with the MSE of other existing estimators i.e $\hat{T}_{yd}^{HH}$, $\hat{T}_{rd}^{HH}$ and $\hat{T}_{zd}^{HH}$. By using (21), (24), (29) and (52), the following conditions are developed

(i) $MSE(\hat{T}_{pd}^{HH}) < MSE(\hat{\tau}_{yd}^{HH})$

$$if \quad \left[\frac{\frac{9}{4}\mu_y^2 + \frac{21}{4}\{\theta(\sigma_y^2+\sigma_u^2) + \theta^*(\sigma_{y(2)}^2 + \sigma_w^2) + \kappa\}}{4\Delta\{\theta\rho_{yx}\sigma_y\sigma_x + \theta^*\rho_{zx(2)}\sigma_z\sigma_{x(2)} - \theta'\rho_{yx}\sigma_y\sigma_x\}}\right] < 1 \tag{53}$$

(ii) $MSE(\hat{T}_{pd}^{HH}) < MSE(\hat{\tau}_{rd}^{HH})$

$$if \quad \left[\frac{\frac{9}{4}\mu_y^2 + \frac{21}{4}\{\theta(\sigma_y^2+\sigma_u^2) + \theta^*(\sigma_{y(2)}^2 + \sigma_w^2) + \kappa\}}{(5\Delta-2G)\{\theta\rho_{yx}\sigma_y\sigma_x + \theta^*\rho_{zx(2)}\sigma_z\sigma_{x(2)} - \theta'\rho_{yx}\sigma_y\sigma_x\}}\right] + \frac{(\Delta^2-G^2)}{(5\Delta-2G)H} < 1 \tag{54}$$

(iii) $MSE(\hat{T}_{pd}^{HH}) < MSE(\hat{\tau}_{zd}^{HH})$

$$if \quad \left[\frac{\frac{9}{4}\mu_y^2 + \frac{21}{4}\{\theta(\sigma_y^2+\sigma_u^2) + \theta^*(\sigma_{y(2)}^2 + \sigma_w^2) + \kappa\}}{(5\Delta-2H)\{\theta\rho_{yx}\sigma_y\sigma_x + \theta^*\rho_{zx(2)}\sigma_z\sigma_{x(2)} - \theta'\rho_{yx}\sigma_y\sigma_x\}}\right] + \frac{(\Delta^2-H^2)}{(5\Delta-2H)H} < 1 \tag{55}$$

For without measurement error, one can obtain the above conditions by assuming $\sigma_u = \sigma_w = 0$.

If the above conditions (53), (54) and (55) holds true, then our proposed estimator is more efficient than other existing estimators respcetively.

6 Simulation Study

It is crucial to look into the cases when our suggested approaches outperform existing ones. So, an empirical investigation of the proposed technique with a data set of natural population and ficitious population is conducted in order to demonstrate the effectiveness of the proposed estimator $\hat{T}_{pd}^{HH}$ in the presence of non-response and measurement error simultaneously using ORRT model over other existing estimators by using R software.

The empirical MSE of the estimator $\hat{T}_{pd}^{HH}$ is computed by

$$EMSE(\hat{T}_{d}^{HH}) = \frac{1}{6000}\sum_{i=1}^{6000}(\hat{T}_{d}^{HH} - \mu_y)^2 \tag{56}$$

where $d = yd, rd, zd, pd$; and μ_y is the population mean of the sensitive study variable.

The percent relative efficiency (PRE) of the estimators $\hat{T}_{d}^{HH}$ with respect to the ordimary mean estimator $\hat{T}_{yd}^{HH}$ is premeditated by using following equation

$$PRE = \frac{MSE(\hat{T}_{yd}^{HH})}{MSE(\hat{T}_{d}^{HH})} \times 100 \tag{57}$$

Now, the unified measure δ as defined in Gupta et al. (2018) is given by

$$\delta = \frac{MSE(\hat{T}_{d}^{HH})}{\Omega_{DP}}. \tag{58}$$

6.1 Natural population Data Set

For real data set, we use Narcotics Control Bureau, New Delhi data which is related to Crime in India 2016 from National Crime Records Bureau (Ministry of Home Affairs), Government of India to elucidate the efficacious performance of our proposed estimator. The population consists of $N = 13$ categories. Further, let X denotes the number of persons arrested and Y denotes the total number of drugs seized as per

Narcotics Control Bureau during 2016 in the country. In order to minimize the impact of scrambling variables on the actual data, we have considered $\mu_T = 1; \sigma_T^2 = 5$ and $\mu_s = 0; \sigma_s^2 = 1$, then we compute the percent relative efficiencies (PREs) with respect to the ordinary mean estimator $\hat{T}_{yd}^{HH}$ of population mean for different probabilities (P) and also evaluated the Privacy and Efficiency (i.e. Unified Measure) as suggested by Gupta et al. (2018). The findings are displayed in Table 1.

Table 1: Percent relative efficiency (PRE) and Unified Measure (δ) of the ORRT Estimators when $f = 2$.

P	PRE				Unified Measure (δ)			
	$\hat{T}_{yd}^{HH}$	$\hat{T}_{rd}^{HH}$	$\hat{T}_{zd}^{HH}$	$\hat{T}_{pd}^{HH}$	$\hat{T}_{yd}^{HH}$	$\hat{T}_{rd}^{HH}$	$\hat{T}_{zd}^{HH}$	$\hat{T}_{pd}^{HH}$
0.2	100.0000	89.0068	106.2112	184.0140	0.2390	0.2686	0.2251	0.1299
0.4	100.0000	92.6382	103.9187	181.9404	0.3708	0.4002	0.3568	0.2038
0.6	100.0000	94.9869	103.4800	181.8529	0.2398	0.2525	0.2318	0.1319
0.8	100.0000	95.5924	103.0160	181.3806	0.275	0.2882	0.2674	0.1518
1.0	100.0000	103.2333	101.8732	169.7269	3.0298	2.9349	2.9741	1.7851

It is visualized from Table 1 that for different values of P, the proposed estimator $\hat{T}_{pd}^{HH}$ are highly efficient with respect to other existing estimator(s) i.e. ($\hat{T}_{yd}^{HH}$, $\hat{T}_{rd}^{HH}$ and $\hat{T}_{zd}^{HH}$). Also, with the increase in value of P, the percent relative efficiency of the estimators $\hat{T}_{zd}^{HH}$ and $\hat{T}_{pd}^{HH}$ decreases and the percent relative efficiency of $\hat{T}_{rd}^{HH}$ increases. Moreover, we also observed that the proposed estimator is always efficient than the other conventional estimator(s) under two-phase sampling.

6.2 Population generation through simulation using normal distribution

Let the scrambling variable S is taken to be a normal distribution with mean equal to zero and varying variance ($0.3^*\sigma_x^2, 0.6^*\sigma_x^2, 1^*\sigma_x^2$) and other scrambling variable T is also confiscate to be a normal distribution with mean one and varying variances $(0.2, 0.5, 1)$. The measurement error of X have a normal distribution with mean zero in both phases; the measurement error of Y in the first phase and Z in the second phase have a normal distribution with mean zero and varying variances $(0, 5, 10)$. Now, we consider a finite population of size 6000 units generated from bivariate normal distribution with means and covariance of (Y, X) as given below.

$$\textbf{Population:} \quad \mu = \begin{bmatrix} -0.02 \\ -0.05 \end{bmatrix}, \ \Sigma = \begin{bmatrix} 13 & 3 \\ 3 & 1 \end{bmatrix}, \ \rho = 0.9$$

From here, $\mu_x = -0.02, \sigma_x^2 = 1, \mu_y = -0.05, \sigma_y^2 = 13, \rho_{yx} = 0.9$ (I)
$\mu_x = -0.0198, \sigma_x^2 = 0.9861, \mu_y = -0.0516, \sigma_y^2 = 13.15575, \rho_{yx} = 0.9605$ (II)

Our parameter values for the set of 6000 data points we generated using R are very close to those in (I), although not exact. So, for the simulation study, we use parameter values in (II).

From population, we consider sample of size $n' = 700$ and assume that the response rate is 40% in the first phase. From n' we take sample of size $n = 600$. This means in the first phase only $240(n_1)$ respondents provide a response to the survey question and $360(n_2)$ of them do not. In the second phase, we take another sample $\left(n_s = \frac{n_2}{f}\right)$ from non-respondent group by using $f = 2, 3, 4$, respectively. We also consider different response rates of 20%, 30%, 40% compared in the simulation study and results are averaged over 6,000 iterations. The results are shown in Tale 2 to 6.

Table 2: Theoretical **(boldface)** and Empirical MSEs/PREs (with/without measurement error) of the ORRT Estimators when Response Rate $= 40\%$, $\sigma_u^2 = \sigma_w^2 = \sigma_v^2 = 1$, $f = 2$ and $\sigma_s^2 = 0.3^* \sigma_x^2$.

Estimator(s)	P	σ_T^2	MSE		PRE		δ
			With M.E.	Without M.E.	With M.E	Without M.E.	
$\hat{T}_{yd}^{HH}$	0.6	0.2	**0.3576**	**0.3546**	**100.0000**	**100.0000**	**0.0020**
			0.3573	0.3543	100.0000	100.0000	0.0021
		0.5	**0.3621**	**0.3591**	**100.0000**	**100.0000**	**0.0019**
			0.3619	0.3589	100.0000	100.0000	0.0020
		1	**0.3789**	**0.3759**	**100.0000**	**100.0000**	**0.0019**
			0.3786	0.3756	100.0000	100.0000	0.0020
	0.8	0.2	**0.4753**	**0.4723**	**100.0000**	**100.0000**	**0.0026**
			0.4512	0.4720	100.0000	100.0000	0.0027
		0.5	**0.4810**	**0.4780**	**100.0000**	**100.0000**	**0.0026**
			0.4808	0.4778	100.0000	100.0000	0.0027
		1	**0.5033**	**0.5003**	**100.0000**	**100.0000**	**0.0026**
			0.5031	0.5001	100.0000	100.0000	0.0027
	1	0.2	**0.6603**	**0.6573**	**100.0000**	**100.0000**	**0.0037**
			0.6601	0.6571	100.0000	100.0000	0.0038
		0.5	**0.6680**	**0.6650**	**100.0000**	**100.0000**	**0.0037**
			0.6677	0.6647	100.0000	100.0000	0.0038
		1	**0.6987**	**0.6957**	**100.0000**	**100.0000**	**0.0037**
			0.6985	0.6955	100.0000	100.0000	0.0038
$\hat{T}_{rd}^{HH}$	0.6	0.2	**0.3318**	**0.3085**	**107.7622**	**114.9412**	**0.0017**
			0.3332	0.3293	107.2499	107.5949	0.0019
		0.5	**0.3374**	**0.3141**	**107.3057**	**107.2185**	**0.0017**
			0.3375	0.3340	107.2185	107.4374	0.0019
		1	**0.3559**	**0.3326**	**106.4404**	**113.0056**	**0.0020**
			0.3545	0.3511	106.7965	107.0006	0.0019
	0.8	0.2	**0.4541**	**0.4308**	**104.6566**	**109.6293**	**0.0025**
			0.4512	0.4477	105.2801	105.4284	0.0026
		0.5	**0.4602**	**0.4368**	**104.5318**	**109.4289**	**0.0024**
			0.4570	0.4535	105.2031	105.3489	0.0026
		1	**0.4829**	**0.4596**	**104.2150**	**108.8535**	**0.0025**
			0.4793	0.4759	104.9447	105.0817	0.0026
	1	0.2	**0.6396**	**0.6162**	**103.2445**	**106.6629**	**0.0035**
			0.6363	0.6328	103.7346	103.8310	0.0037
		0.5	**0.6470**	**0.6236**	**103.2403**	**106.6267**	**0.0035**
			0.6439	0.6204	103.6968	103.7918	0.0037
		1	**0.6775**	**0.6541**	**103.1377**	**106.3584**	**0.0035**
			0.6747	0.6512	103.5345	103.6243	0.0036

		0.2	**0.3464**	**0.3231**	**103.2334**	**109.7206**	**0.0018**
			0.3461	0.3235	103.2335	109.7066	0.0020
	0.6	0.5	**0.3515**	**0.3295**	**103.0017**	**108.9697**	**0.0018**
			0.3513	0.3293	103.0018	108.9827	0.0020
		1	**0.3693**	**0.3490**	**102.5933**	**107.6857**	**0.0018**
			0.3691	0.3488	102.5939	107.6969	0.0020
		0.2	**0.4666**	**0.4481**	**101.8477**	**105.3896**	**0.0026**
			0.4664	0.4479	101.8481	105.3981	0.0027
$\hat{T}_{zd}^{HH}$	0.8	0.5	**0.4726**	**0.4543**	**101.7924**	**105.2225**	**0.0025**
			0.4723	0.4541	101.7928	105.2307	0.0027
		1	**0.4951**	**0.4772**	**101.6591**	**104.8206**	**0.0025**
			0.4948	0.4771	101.6595	104.8283	0.0027
		0.2	**0.6519**	**0.6337**	**101.2961**	**103.7286**	**0.0036**
			0.6517	0.6336	101.2966	103.7349	0.0038
	1	0.5	**0.6594**	**0.6410**	**101.2899**	**103.7104**	**0.0036**
			0.6592	0.6408	101.2904	103.7167	0.0038
		1	**0.6900**	**0.6714**	**101.2587**	**103.6176**	**0.0036**
			0.6898	0.6712	101.2592	103.6238	0.0037
		0.2	**0.1931**	**0.0480**	**186.3887**	**737.8221**	**0.0052**
			0.1865	0.0418	192.4526	846.1338	0.0051
	0.6	0.5	**0.1972**	**0.0594**	**185.1398**	**604.4946**	**0.0053**
			0.1906	0.0532	191.5713	674.5225	0.0053
		1	**0.2032**	**0.0765**	**183.6343**	**491.4042**	**0.0054**
			0.1967	0.0702	189.7926	534.6927	0.0053
		0.2	**0.2092**	**0.0932**	**228.8493**	**506.5725**	**0.0055**
			0.2027	0.0869	234.3352	542.8536	0.0055
$\hat{T}_{pd}^{HH}$	0.8	0.5	**0.2102**	**0.0958**	**227.1287**	**498.5341**	**0.0065**
			0.2036	0.0896	245.0796	533.2080	0.0065
		1	**0.2118**	**0.1003**	**236.0785**	**498.3858**	**0.0065**
			0.2052	0.0941	237.6249	531.4495	0.0065
		0.2	**0.2239**	**0.1105**	**294.8411**	**594.5054**	**0.0056**
			0.2174	0.1042	303.6081	630.1460	0.0056
	1	0.5	**0.2231**	**0.1081**	**299.4120**	**614.8670**	**0.0056**
			0.2165	0.1018	308.3517	652.5821	0.0055
		1	**0.2222**	**0.1057**	**323.8602**	**658.1059**	**0.0065**
			0.2156	0.0994	341.4280	699.4517	0.0064

Table 3: Theoretical **(boldface)** and Empirical MSEs/PREs (with/without measurement error) of the ORRT Estimators when Response Rate $= 40\%$, $\sigma_u^2 = \sigma_w^2 = \sigma_v^2 = 1$, $f = 2$ and $\sigma_s^2 = 0.6^{*}\sigma_x^2$.

Estimator(s)	P	σ_T^2	MSE		PRE		δ
			With M.E.	Without M.E.	With M.E	Without M.E.	
$\hat{T}_{yd}^{HH}$	0.6	0.2	**0.3580**	**0.3550**	**100.0000**	**100.0000**	**0.0020**
			0.3578	0.3548	100.0000	100.0000	0.0021
		0.5	**0.3626**	**0.3596**	**100.0000**	**100.0000**	**0.0020**
			0.3623	0.3593	100.0000	100.0000	0.0020
		1	**0.3794**	**0.3764**	**100.0000**	**100.0000**	**0.0020**
			0.3791	0.3761	100.0000	100.0000	0.0021
	0.8	0.2	**0.4759**	**0.4729**	**100.0000**	**100.0000**	**0.0027**
			0.4757	0.4727	100.0000	100.0000	0.0028
		0.5	**0.4817**	**0.4787**	**100.0000**	**100.0000**	**0.0027**
			0.4815	0.4785	100.0000	100.0000	0.0028
		1	**0.5039**	**0.5009**	**100.0000**	**100.0000**	**0.0027**
			0.5037	0.5007	100.0000	100.0000	0.0027
	1	0.2	**0.6612**	**0.6582**	**100.0000**	**100.0000**	**0.0038**
			0.6609	0.6579	100.0000	100.0000	0.0038
		0.5	**0.6688**	**0.6658**	**100.0000**	**100.0000**	**0.0038**
			0.6686	0.6656	100.0000	100.0000	0.0038
		1	**0.6996**	**0.6966**	**100.0000**	**100.0000**	**0.0037**
			0.6993	0.6963	100.0000	100.0000	0.0038
$\hat{T}_{rd}^{HH}$	0.6	0.2	**0.3321**	**0.3088**	**107.7956**	**114.9692**	**0.0019**
			0.3333	0.3298	107.3674	107.5907	0.0019
		0.5	**0.3378**	**0.3144**	**107.3387**	**114.3500**	**0.0019**
			0.3379	0.3345	107.2149	107.4335	0.0019
		1	**0.3563**	**0.3330**	**106.4717**	**113.0324**	**0.0019**
			0.3550	0.3515	106.7935	106.9973	0.0019
	0.8	0.2	**0.4546**	**0.4313**	**104.6900**	**109.6589**	**0.0026**
			0.4518	0.4484	105.2786	105.4267	0.0026
		0.5	**0.4607**	**0.4373**	**104.5648**	**109.4582**	**0.0026**
			0.4576	0.4542	105.2018	105.3473	0.0026
		1	**0.4834**	**0.4601**	**104.2466**	**108.8819**	**0.0026**
			0.4800	0.4765	104.9438	105.0806	0.0026
	1	0.2	**0.6405**	**0.6171**	**103.2320**	**106.6440**	**0.0037**
			0.6372	0.6579	103.7282	103.8244	0.0037
		0.5	**0.6479**	**0.6245**	**103.2269**	**106.6089**	**0.0037**
			0.6448	0.6413	103.6907	103.7855	0.0037
		1	**0.6784**	**0.6550**	**103.1267**	**106.3426**	**0.0036**
			0.6755	0.6720	103.5290	103.6187	0.0037

$\hat{T}_{zd}^{HH}$	0.6	0.2	**0.3468**	**0.3234**	**103.2533**	**109. 7702**	**0.0020**
			0.3465	0.3232	103.2535	109.7843	0.0020
		0.5	**0.3519**	**0.3297**	**103.0207**	**109.0428**	**0.0020**
			0.3517	0.3296	103.0209	109.0297	0.0020
		1	**0.3697**	**0.3493**	**102.6104**	**107.7389**	**0.0019**
			0.3695	0.3491	102.6107	107.7502	0.0020
	0.8	0.2	**0.4672**	**0.4485**	**101.8649**	**105.4418**	**0.0027**
			0.4670	0.4483	101.8654	105.4503	0.0027
		0.5	**0.4731**	**0.4547**	**101.8093**	**105.2735**	**0.0027**
			0.4729	0.4545	101.8097	105.8761	0.0027
		1	**0.4956**	**0.4776**	**101.6750**	**104.2818**	**0.0026**
			0.4954	0.4775	101.6754	104.8684	0.0027
	1	0.2	**0.6528**	**0.6347**	**101.2837**	**103.6923**	**0.0037**
			0.6525	0.6345	101.2842	103.7177	0.0038
		0.5	**0.6603**	**0.6419**	**101.2903**	**103.6985**	**0.0037**
			0.6601	0.6417	101.2908	103.7114	0.0038
		1	**0.6909**	**0.6723**	**101.2536**	**103.6026**	**0.0037**
			0.6907	0.6721	101.2541	103.6087	0.0037
$\hat{T}_{pd}^{HH}$	0.6	0.2	**0.1926**	**0.0466**	**187.0480**	**619.7398**	**0.0052**
			0.1861	0.0404	192.2675	877.3925	0.0051
		0.5	**0.1967**	**0.0580**	**185.8830**	**500.4924**	**0.0063**
			0.1909	0.0518	190.5370	693.4262	0.0062
		1	**0.2028**	**0.0752**	**184.3389**	**461.6448**	**0.0064**
			0.1963	0.0689	193.1477	545.3947	0.0063
	0.8	0.2	**0.2086**	**0.0916**	**228.0659**	**516.1391**	**0.0065**
			0.2021	0.0853	235.3226	553.7833	0.0064
		0.5	**0.2096**	**0.0943**	**229.7793**	**507.6031**	**0.0055**
			0.2031	0.0880	237.0581	543.5286	0.0054
		1	**0.2112**	**0.0988**	**238.5560**	**506.8662**	**0.0065**
			0.2047	0.0925	246.0602	541.0425	0.0064
	1	0.2	**0.2241**	**0.1110**	**294.9823**	**592.5626**	**0.0076**
			0.2176	0.1047	303.7465	627.9206	0.0075
		0.5	**0.2232**	**0.1086**	**299.5632**	**612.9584**	**0.0066**
			0.2167	0.1023	308.5007	650.3876	0.0065
		1	**0.2223**	**0.1061**	**314.5889**	**656.2424**	**0.0055**
			0.2158	0.0998	324.0195	697.3000	0.0054

Table 4: Theoretical **(boldface)** and Empirical MSEs/PREs (with/without measurement error) of the ORRT Estimators when Response Rate $= 40\%$, $\sigma_u^2 = \sigma_w^2 = \sigma_v^2 = 1$, $f = 2$ and $\sigma_s^2 = 1^* \sigma_x^2$.

Estimator(s)	P	σ_T^2	MSE		PRE		δ
			With M.E.	Without M.E.	With M.E	Without M.E.	
$\hat{T}_{yd}^{HH}$	0.6	0.2	**0.3592**	**0.3562**	**100.0000**	**100.0000**	**0.0021**
			0.3590	0.3560	100.0000	100.0000	0.0020
		0.5	**0.3637**	**0.3607**	**100.0000**	**100.0000**	**0.0021**
			0.3635	0.3605	100.0000	100.0000	0.0021
		1	**0.3805**	**0.3775**	**100.0000**	**100.0000**	**0.0020**
			0.3803	0.3773	100.0000	100.0000	0.0021
	0.8	0.2	**0.4775**	**0.4745**	**100.0000**	**100.0000**	**0.0027**
			0.4773	0.4743	100.0000	100.0000	0.0028
		0.5	**0.4833**	**0.4803**	**100.0000**	**100.0000**	**0.0027**
			0.4831	0.4801	100.0000	100.0000	0.0028
		1	**0.5055**	**0.5025**	**100.0000**	**100.0000**	**0.0027**
			0.5053	0.5023	100.0000	100.0000	0.0028
	1		**0.6632**	**0.6602**	**100.0000**	**100.0000**	**0.0038**
			0.6630	0.6600	100.0000	100.0000	0.0039
		0.5	**0.6709**	**0.6679**	**100.0000**	**100.0000**	**0.0038**
			0.6706	0.6676	100.0000	100.0000	0.0039
		1	**0.7016**	**0.6986**	**100.0000**	**100.0000**	**0.0037**
			0.7014	0.6984	100.0000	100.0000	0.0038
$\hat{T}_{rd}^{HH}$	0.6	0.2	**0.3331**	**0.3098**	**107.8265**	**114.9793**	**0.0019**
			0.3344	0.3309	107.3504	107.5727	0.0019
		0.5	**0.3388**	**0.3154**	**107.3700**	**114.3616**	**0.0019**
			0.3391	0.3356	107.1987	107.4163	0.0020
		1	**0.3573**	**0.3339**	**106.5031**	**113.0465**	**0.0019**
			0.3561	0.3527	106.7794	106.9824	0.0019
	0.8	0.2	**0.4560**	**0.4326**	**104.7265**	**109.6816**	**0.0026**
			0.4534	0.4499	105.2681	105.4156	0.0027
		0.5	**0.4620**	**0.4387**	**104.6012**	**109.4813**	**0.0026**
			0.4592	0.4557	105.1917	105.3367	0.0026
		1	**0.4848**	**0.4614**	**104.2820**	**108.9053**	**0.0026**
			0.4815	0.4781	104.9351	104.9246	0.0026
	1	0.2	**0.6426**	**0.6193**	**103.2102**	**106.6085**	**0.0037**
			0.6392	0.6358	103.7142	103.8100	0.0037
		0.5	**0.6500**	**0.6267**	**103.2041**	**106.5748**	**0.0037**
			0.6468	0.6434	103.6771	103.7716	0.0037
		1	**0.6805**	**0.6571**	**103.1074**	**106.3123**	**0.0036**
			0.6776	0.6741	103.5168	103.6061	0.0037

		0.2	**0.3478**	**0.3243**	**103.2744**	**109. 8371**	**0.0020**
			0.3476	0.3241	103.2746	109.8514	0.0018
	0.6	0.5	**0.3530**	**0.3306**	**103.0410**	**109.0937**	**0.0020**
			0.3528	0.3304	103.0412	109.1070	0.0019
		1	**0.3708**	**0.3502**	**102.6292**	**107.7972**	**0.0019**
			0.3705	0.3500	102.6295	107.8087	0.0020
		0.2	**0.4687**	**0.4497**	**101.8850**	**105.5023**	**0.0027**
			0.4685	0.4496	101.8855	105.5110	0.0026
$\hat{T}_{zd}^{HH}$	0.8	0.5	**0.4746**	**0.4559**	**101.8291**	**105.3329**	**0.0027**
			0.4744	0.4558	101.8295	105.3413	0.0026
		1	**0.4971**	**0.4789**	**101.6938**	**104.9246**	**0.0026**
			0.4969	0.4787	101.6942	104.9324	0.0027
		0.2	**0.6549**	**0.6369**	**101.2735**	**103.6621**	**0.0037**
			0.6547	0.6367	101.2810	103.6683	0.0038
	1	0.5	**0.6624**	**0.6441**	**101.2806**	**103.6826**	**0.0037**
			0.6622	0.6439	101.2740	103.6888	0.0038
		1	**0.6930**	**0.6745**	**101.2449**	**103.5770**	**0.0037**
			0.6928	0.6743	101.2454	103.5831	0.0038
		0.2	**0.1919**	**0.0447**	**188.1889**	**796.5094**	**0.0052**
			0.1854	0.0385	193.6003	923.4392	0.0052
	0.6	0.5	**0.1960**	**0.0562**	**187.1486**	**641.9381**	**0.0053**
			0.1895	0.0500	191.8084	721.0136	0.0052
		1	**0.2022**	**0.0734**	**185.5478**	**513.7317**	**0.0054**
			0.1957	0.0672	194.3453	560.9757	0.0053
		0.2	**0.2079**	**0.0894**	**237.0130**	**530.2706**	**0.0065**
			0.2014	0.0832	240.1500	569.9182	0.0064
$\hat{T}_{pd}^{HH}$	0.8	0.5	**0.2088**	**0.0921**	**231.3774**	**520.9898**	**0.0065**
			0.2023	0.0859	238.7344	558.7509	0.0064
		1	**0.2105**	**0.0967**	**229.6770**	**519.3552**	**0.0065**
			0.2039	0.0904	247.7318	555.1562	0.0064
		0.2	**0.2243**	**0.1117**	**295.5798**	**590.7291**	**0.0076**
			0.2178	0.1054	304.3522	625.7532	0.0076
	1	0.5	**0.2235**	**0.1092**	**300.1723**	**611.1642**	**0.0056**
			0.2169	0.1029	309.1188	648.2540	0.0055
		1	**0.2226**	**0.1067**	**315.2079**	**654.4969**	**0.0065**
			0.2160	0.1004	324.6483	695.2105	0.0065

The above tables i.e Table 2, Table 3 and Table 4 represents the theoretical and empirical MSEs and PREs of the ORRT estimators when all the variances of measurement errors on X, Y and Z ($\sigma_u^2, \sigma_w^2, \sigma_w^2$) are equal to 1 and the variance of scrambling variable S is equal to ($0.3^*\sigma_x^2, 0.6^*\sigma_x^2, 1^*\sigma_x^2$) with response rate set equal to 40%, respectively.

Finally, it is also clear that the proposed estimator ($\hat{T}_{pd}^{HH}$) performing well as compared to the other considered mean estimators i.e the ordinary RRT mean estimator ($\hat{T}_{yd}^{HH}$), the ratio estimator ($\hat{T}_{rd}^{HH}$) and Zhang et al. (2021) mean estimator ($\hat{T}_{zd}^{HH}$) even when very large measurement errors are present. The same is true for almost all cases even when measurement errors are not present.

Table 5: Theoretical (**boldface**) and Empirical MSEs/PREs (with/without measurement error) of the ORRT Estimators when $\sigma_u^2 = \sigma_w^2 = \sigma_v^2 = 1, 5, 10$, $\sigma_T^2 = 0.5$, $P = 1$ and $\sigma_s^2 = 0.6^*\sigma_x^2$.

Estimator(s)	**f**	**MSE**			**PRE**		
		$W = 1$	$\sigma_T^2 = 0.5$	$W_2 = 40\%$	$W = 1$	$\sigma_T^2 = 0.5$	$W_2 = 40\%$
		1	5	10	1	5	10
$\hat{T}_{yd}^{HH}$	2	**0.4833**	**0.4953**	**0.5103**	**100.0000**	**100.0000**	**100.0000**
		0.4831	0.4951	0.5101	100.0000	100.0000	100.0000
	3	**0.7866**	**0.8046**	**0.8271**	**100.0000**	**100.0000**	**100.0000**
		0.7863	0.8043	0.8268	100.0000	100.0000	100.0000
	4	**0.3305**	**0.5545**	**0.6840**	**100.0000**	**100.0000**	**100.0000**
		1.0302	1.0542	1.0842	100.0000	100.0000	100.0000
$\hat{T}_{rd}^{HH}$	2	**0.4620**	**0.5554**	**0.6721**	**104.6012**	**103.4251**	**101.9015**
		0.4592	0.4731	0.4905	105.1191	104.6332	103.9797
	3	**0.8045**	**0.9445**	**0.9096**	**100.4560**	**100.2551**	**100.3512**
		0.7657	0.7866	0.8127	102.6898	102.2533	101.7370
	4	**0.7190**	**0.6057**	**0.7391**	**102.7781**	**102.3303**	**101.8035**
		1.0024	1.0302	1.0650	101.1232	101.1235	100.9950
$\hat{T}_{zd}^{HH}$	2	**0.4746**	**0.4929**	**0.5090**	**101.8291**	**100.4930**	**100.5212**
		0.4744	0.4926	0.5088	101.8295	100.4931	100.5213
	3	**0.7855**	**0.8042**	**0.8269**	**100.1404**	**100.0415**	**100.0216**
		0.7852	0.8040	0.8267	100.1406	100.0417	100.0218
	4	**0.5159**	**0.6499**	**0.6820**	**101.4360**	**100.4344**	**100.2267**
		1.0157	1.0497	1.0818	101.4359	100.0216	100.0314
$\hat{T}_{pd}^{HH}$	2	**0.2088**	**0.3229**	**0.4239**	**231.8854**	**153.3638**	**120.3838**
		0.2023	0.3164	0.4173	238.7344	156.4748	122.2237
	3	**0.4444**	**0.6570**	**0.8577**	**196.8261**	**155.6447**	**125.5962**
		0.4378	0.5103	0.8511	200.0631	157.6073	126.8290
	4	**0.3996**	**0.6504**	**0.6585**	**231.3774**	**160.4855**	**126.4372**
		0.3930	0.5169	0.6519	235.3106	162.0762	127.3894

Table 5 represents the theoretical and empirical MSEs and PREs of the ORRT estimator under different variances of measurement errors $(0, 5, 10)$ when the sensitivity level P is 1, variance of T is 0.5 and response rate is 40%. In this table, the MSE of each mean estimator increases with increase in measurement errors from 1 to 5. For example, when the variance of measurement errors increased from 0 to 10 and the value of f is 2 then the MSE of the proposed estimator $\hat{T}_{pd}^{HH}$ increased from 0.2088 to 0.4239 and same in case when the value of $f = 3$ and 4.

In the above table, we see that our proposed estimator ($\hat{T}_{pd}^{HH}$) is more efficient than the other existing estimators i.e usual unbiased estimator ($\hat{T_{yd}^{HH}}$), Ratio estimator ($\hat{T}_{rd}^{HH}$) and Zhang et al. (2021) mean estimator ($\hat{T}_{zd}^{HH}$). The same is true for almost all cases even when measurement errors are not present.

Table 6: Theoretical **(boldface)** and Empirical MSEs/PREs (with/without measurement error) of the ORRT Estimators under the conditions of Response Rate (RR)= $20\%, 30\%, 40\%$ when $\sigma_u^2 = \sigma_w^2 = \sigma_v^2 = 1$, $\sigma_T^2 = 0.5$, $P = 0.8$ and $\sigma_s^2 = 0.6^*\sigma_x^2$.

Estimator(s)	**f**	**MSE**			**PRE**		
		20%	30%	40%	20%	30%	40%
$\hat{T}_{yd}^{HH}$	2	**0.4617**	**0.4729**	**0.4817**	**100.0000**	**100.0000**	**100.0000**
		0.4615	0.4727	0.4815	100.0000	100.0000	100.0000
	3	**0.7276**	**0.7641**	**0.7842**	**100.0000**	**100.0000**	**100.0000**
		0.7273	0.7639	0.7839	100.0000	100.0000	100.0000
	4	**0.9055**	**1.0712**	**1.0273**	**100.0000**	**100.0000**	**100.0000**
		0.9053	1.0710	1.0270	100.0000	100.0000	100.0000
$\hat{T}_{rd}^{HH}$	2	**0.4302**	**0.4528**	**0.4607**	**107.5504**	**104.9610**	**104.5648**
		0.4361	0.4490	0.4576	105.8210	105.2680	105.2018
	3	**0.6765**	**0.7280**	**0.8027**	**107.3113**	**104.4533**	**100.9981**
		0.6962	0.7350	0.7634	104.4673	103.9334	102.6865
	4	**0.8257**	**1.0864**	**1.0171**	**107.1415**	**101.4265**	**101.6512**
		0.8675	1.0472	0.9994	104.3573	102.2682	102.7665
$\hat{T}_{zd}^{HH}$	2	**0.4469**	**0.4648**	**0.4731**	**107.1415**	**103.0637**	**101.8093**
		0.4467	0.4646	0.4729	107.1410	101.7387	101.8097
	3	**0.6941**	**0.7414**	**0.7831**	**104.8157**	**101.7391**	**101.3835**
		0.6939	0.7412	0.7839	104.8163	103.0633	101.3834
	4	**0.8451**	**1.0661**	**1.0132**	**103.3138**	**100.4836**	**100.1302**
		0.8449	1.0658	1.0130	103.3145	100.4838	100.1306
$\hat{T}_{pd}^{HH}$	2	**0.1656**	**0.2104**	**0.2096**	**402.0584**	**292.2670**	**229.7793**
		0.1591	0.2039	0.2031	416.8889	299.6558	237.0581
	3	**0.1809**	**0.2614**	**0.4001**	**278.7902**	**224.7008**	**229.3273**
		0.1744	0.2549	0.3935	290.0442	231.7888	232.6872
	4	**0.1936**	**0.5235**	**0.4479**	**796.6365**	**204.6071**	**195.9773**
		0.1072	0.5169	0.4413	844.4984	207.1690	199.1960

Table 6 represents the theoretical and empirical MSEs and PREs of the ORRT estimators under different response rates when the variance of measurement errors is equal to 1, sensitivity level P is equal to 0.8, and variance of T is equal to 0.5. It is envisaged from above table that the increase in 'f', there is an increase in the value of MSE of all the considered estimators. And for different response rate the MSE of the proposed estimator first increase and then decreased for $f = 2, 3, 4$.

Overall, our proposed estimator is performing good than the other considered estimators i.e ordinary mean estimator ($\hat{T}_{yd}^{HH}$), Ratio estimator ($\hat{T}_{rd}^{HH}$) and Zhang et al. (2021) mean estimator ($\hat{T}_{zd}^{HH}$). The same is true for almost all cases even when measurement errors are not present.

7 Conclusion

This article considered a problem for the estimation of sensitive variable in the presence of non-response and measurement errors simultaneously in two-phase sampling using ORRT models.The expressions of bias and minimum MSE of the proposed estimator ($\hat{T}_{pd}^{HH}$) has been obtained to the second order approximation. For empirical results, we studied the real-world application and model-based simulation and obtained the percent relative efficiency for different response rates The properties of the proposed estimator have also been obtained. The relative performance of the proposed estimator with other considered estimators i.e ordinary mean estimator ($\hat{T}_{yd}^{HH}$), Ratio estimator ($\hat{T}_{rd}^{HH}$) and Zhang et al. (2021) mean estimator ($\hat{T}_{zd}^{HH}$) is also discussed and the conditions have obtained. It is also clear from the theoretical conditions that the performance of the proposed estimator is better than the other existing estimators. Thus, our recommendation is to prefer our proposed estimator to the survey statisticians for their application in real life population.

Acknowledgement

The authors appreciate the knowledgeable referee's comments, which dramatically improved the paper's excellence.

References

1. Cochran, W. G. (1968), 'Errors of measurement in statistics', *Technometrics,* 10(4), 637–666, DOI: 10.2307/1267450.

2. Cochran, W. G. (1977), *Sampling Techniques*, 3rd edn, Wiley, New York.

3. Diana, G. and Perri, P. F. (2011), 'A class of estimators for quantitative sensitive data, statistical papers', *Statistical Papers,* 52, 633–650, DOI: 10.1007/s00362–009–0273–1.

4. Diana, G., Riaz, S. and Shabbir, J. (2014), 'Hansen and hurwitz estimator with scrambled response on the second call', *Journal of Applied Statistics,* 41(3), 596–611, DOI: 10.1080/02664763.2013.846305.

5. Eichhorn, B. H. and Hayre, L. S. (1983), 'Scrambled randomized response methods for obtaining sensitive quantitative data', *Journal of Statistical Planning and Inference,* 7(4), 307–316, DOI: 10.1016/0378–3758(83)90002–2.

6. Guha, S. and Chandra, H. (2019), 'Improved estimation of finite population mean in two-phase sampling with sub-sampling of the non-respondents', *Mathematical Population Studies,* 28(1), 24–44, DOI: 10.1080/08898480.2019.1694325.

7. Gupta, S., Gupta, B. and Singh, S. (2002), 'Estimation of sensitivity level of personal interview survey questions', *Journal of Statistical Planning and inference,* 100, 239–247, DOI: 10.1016/S0378–3758(01)00137–9.

8. Gupta, S., Kalucha, J., Shabbir, J. and Dass, B. K. (2014), 'Estimation of finite population mean using optional rrt models in the presence of non-sensitive auxiliary information', *American Journal of Mathematical and Management Sciences,* 33(2), 147–159, DOI: 10.1080/01966324.2014.908332.

9. Gupta, S., Mehta, S., Shabbir, J. and Khalil, S. (2018), 'A unified measure of respondent privacy and model efficiency in quantitative rrt models', *Journal of Statistical Theory and Practice,* 12(3), 506–511.

10. Gupta, S., Shabbir, J., Sousa, R. and Corte-Real, P. (2012), 'Estimation of the mean of a sensitive variable in the presence of auxiliary information', *Communications in Statistics-Theory and Methods,* 41, 13–14, DOI: 10.1080/03610926.2011.641654.

11. Hansen, M. H. and Hurwitz, W. N. (1946), 'The problem of non-response in sample surveys', *Journal of the American Statistical Association,* 41(236), 517–529, DOI: 10.1080/01621459.1946.10501894.

12. Khalil, S., Amin, M. and Hanif, M. (2018), 'Estimation of population mean for a sensitive variable in the presence of measurement error', *Journal of Statistics and Management Systems,* 21(1), 81–91, DOI: 10.1080/09720510.2017.1367478.

13. Khalil, S., Zhang, Q. and Gupta, S. (2021), 'Mean estimation of sensitive variables under measurement errors using optional rrt models', *Communications in Statistics–Simulation and Computation,* **50**(5), 1417–1426, DOI: 10.1080/03610918.2019.1584298.

14. Kumar, S. and Bhougal, S. (2011), 'Estimation of the population mean in presence of non-response', *Communications of the Korean statistical society,* **18**(1), 1–12, DOI: 10.5351/CKSS.2011.18.4.537.

15. Kumar, S. and Bhougal, S. (2018), 'Study on non response and measurement error using double sampling scheme', *Journal of Statistics Applications & Probability Letters,* 5(1), 43–52, DOI: 10.18576/jsapl/050105.

16. Misra, P., Tiwari, N. and Kaur, T. (2021), 'An enhanced two phase sampling ratio estimator for estimating population mean', *Journal of Scientific Research Institute of Science,* 65(3), 1–16, DOI: 10.37398/JSR.2021.650321.

17. Mushtaq, N. and Amin, M. (2020), 'Joint influence of double sampling and randomized response technique on estimation method of mean', *Applied Mathematics,* 10(1), 12–19, DOI: 10.5923/j.am.20201001.03.

18. Neyman, J. (1938), 'Contribution to the theory of sampling human populations', *Journal of the American Statistical Association,* 33(201), 101–116, DOI: 10.1080/01621459.1938.10503378.

19. Pollock, K. and Bek, Y. (1976), 'A comparison of three randomized response models for quantitative data', *Journal of the American Statistical Association,* 71(356), 884–886, DOI: 10.1080/01621459.1976.10480963.

20. Sanaullah, A., Ali, H., Noor-ul Amin, M. and Hanif, M. (2014), 'Generalized exponential chain ratio estimators under stratified two-phase random sampling', *Applied Mathematics and Computation,* 226, 541–547, DOI: 10.1016/j.amc.2013.10.088.

21. Singh, H. P. and Karpe, N. (2010), 'Estimation of mean, ratio and product using auxiliary information in the presence of measurement errors in sample surveys', *Journal of Statistical Theory and Practice,* 4(1), 111–136, DOI: 10.1080/15598608.2010.10411976.

22. Singh, H. P. and Vishwakarma, G. K. (2007), 'Modified exponential ratio and product estimators for finite population mean in double sampling', *Austrian Journal of Statistics,* 36(3), 217–225, DOI: 10.17713/ajs.v36i3.333.

23. Singh, V. K., Singh, R. and Smarandache, F. (2014), 'Different-type estimators for estimation of mean in the presence of measurement error', *arXiv preprint, DOI: 10.48550/arXiv.1410.0279* .

24. Sukhatme, B. V. (1962), 'Some ratio-type estimators in two-phase sampling', *Journal of the American Statistical Association,* 57(299), 628–632, DOI: 10.1080/01621459.1962.10500551.

25. Sukhatme, P. and Sukhatme, B. (1970), *Sampling theory of surveys with applications*, Rome, Second Edition, DOI: 10.2307/2333452.

26. Warner, S. L. (1965), 'Randomized response: a survey technique for eliminating evasive answer bias', *Journal of the American Statistical Association,* 60(309), 63–69, DOI: 10.1080/01621459.1965.10480775.

27. Yan, Z., Wang, J. and Lai, J. (2008), 'An efficiency and protection degree-based comparison among the quantitative randomized response strategies', *Communications in Statistics–Theory and Methods,* 38(3), 400–408, DOI: 10.1080/03610920802220785.

28. Zahid, E. and Shabbir, J. (2019), 'Estimation of finite population mean for a sensitive variable using dual auxiliary information in the presence of measurement errors', *PLOS ONE,* 14(2), DOI: 10.1371/journal.pone.0212111.

29. Zaman, T. and Kadilar, C. (2019), 'New class of exponential estimators for finite population mean in two-phase sampling', *Communications in Statistics–Theory and Methods,* 50, 1–16, DOI: 10.1080/03610926.2019.1643480.

30. Zhang, Q., Kalucha, G., Gupta, S. and Khalil, S. (2018), 'Ratio estimation of the mean under rrt models', *Journal of Statistics and Managements Systems,* 22(1), 97–113, DOI: 10.1080/09720510.2018.1533513.

31. Zhang, Q., Khalil, S. and Gupta, S. (2021), 'Mean estimation of sensitive variables under non-response and measurement errors using optional rrt models', *Journal of Statistical Theory and Practice,* 15(3), 1–15, DOI: 10.1007/s42519–020–00135–2.